Fast Charging Station Placement With Elastic Demand

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***Abstract*—The scarcity of efficient charging infrastructures has suppressed penetration rate of Electric Vehicles (EVs). This paper mainly focuses on the Fast Charging Station (FCS) placement problem, especially with the elastic demand. We first propose the distance preference and waiting time preference to capture the elastic EV charging demand. Moreover, a fixed-point equation is proposed to illustrate the relationship between serving demand rate and waiting time at station. We further formulate the problem of FCS placement with elastic demand as a bi-level optimization problem. In the upper level, we propose a heuristic algorithm to determine the optimal location of charging stations, with the goal of maximizing overall system profit. In the lower level, queueing theory is utilized to analyze the optimal capacity of each charging station. The sufficient condition for the existence of optimal station capacity is given. Simulation results demonstrate the effectiveness of our approach in improving system profit and reducing demand loss rate.**

I. Introduction

Benefited from recent technology innovation in batteries, EVs are expected to spark an energy revolution in transportation [1]–[3]. Unfortunately, the scarcity of efficient charging infrastructures has suppressed the widespread usage of EVs. As current mainstream charging method, slow charging fed in residential areas or public charging spots, usually takes many hours (6-8 hours) to get EVs fully charged due to low voltage.

Facilitated by powerful commercial charging stations, fast charging together with professional management would be a promising approach [4]–[5]. Its essence lies in exploiting market power to provide Quality of Service (QoS) guaranteed services. However, such potential is still under utilized, especially with the tension between surging charging demand and increasing shortage station capacity. All of this is placing enormous pressure on charging service providers to make reasonable decision for planning commercial charging stations.

The advent of fast charging technology, has enabled more flexibility for EV users to adjust charging strategies, bringing forward higher request to service offerings. Compared to fuel vehicles, EVs usually have shorter driving range and longer "refueling" time [6]–[7]. Such distinctions have effects on two main aspects: (1) the mileage anxiety pushes EV users away from far charging stations [8]; (2) heavily-loaded stations are often less preferred choices for EV users. In particular, EV users' elastic charging demand with respect to travel distance and waiting time at station is highlighted in this paper.

There has been substantial researches on FCS placement problem [9]–[14]. Actually, inappropriate charging station placement will not only bring inconvenience to EV users' daily charging, but also reduce the profit of charging service provider [9]–[10]. In order to minimize social cost, Xiong *et al.* [11] formulated the FCS placement as a bi-level problem, where optimal station allocation is determined by capturing competitive charging behaviors of EV users. However, these works typically either focus on random charging demand case, or separate travel distance from waiting time in elastic demand case. Lam *et al.* [12] considered the mileage anxiety of EV users, and incorporated network flow model into charging station placement problem. But they ignored the capacity of charging stations. Gimeńez *et al.* [13] considered the loss of EV users due to long distance but neglected the effect of waiting time.

To this end, we are inspired to study the FCS placement incorporating the elastic charging demand. It is based on the fact that the charging demand is elastic with respect to distance preference and waiting time preference. In particular, a fixed-point equation is proposed to capture the relationship between serving demand rate and waiting time at station. We further formulate the FCS placement with elastic demand as a bi-level optimization problem. In the upper level, a heuristic algorithm is proposed to determine the optimal location of charging stations by maximizing system profit. In the lower level, the multiple-server queueing model is used to allocate the charging spots number at each station. Moreover, we analyze the sufficient condition for the existence of optimal station capacity.

The contributions of this work are summarized as follows:

- This paper addresses the issue of FCS placement with elastic charging demand. To our knowledge, this is the first paper that introduces elastic charging demand about distance preference and waiting time preference to fast charging network model. Specifically, a fixed-point equation is proposed to describe the relationship between serving demand rate and waiting time at station.
- With elastic charging demand, the FCS placement problem actually falls into nonlinear integer programming problem, where neither desirable nor effective approaches are available. We first prove that the fixed-point

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equation has and only has one root and the Newton-Raphson method is adopted to solve it. After that, a heuristic algorithm is developed to determine the optimal location and capacity of charging stations.
- Simulation results demonstrate the effectiveness of our approach in improving system profit and reducing demand loss rate. In addition, the impacts of EV user number and fix station cost on placement strategies are also illustrated.

The rest of this paper is organized as follows. First we provide the details of system model and formulate a bi-level optimization problem in Section II. In Section III, we analyze the lower-level problem and upper-level problem of FCS placement separately. A heuristic algorithm is proposed to determine the location and capacity of charging stations. In Section IV, we present the numerical results of our approach. Finally, we conclude our work in Section V.

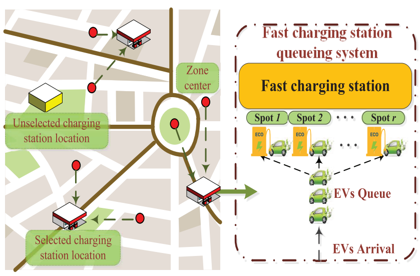

Fig. 1. Illustration of Fast Charging System

II. SYSTEM MODEL AND PROBLEM FORMULATION

The details of fast charging system are illustrated in this section. We first introduce service region topology and charging demand model. After that, the problem of FCS placement with elastic charging demand is presented.

A. Demand Zones and Charging Stations

Consider the general fast charging system consisting of EV users and charging service provider. The provider is responsible for providing charging services for EV users in service region G. We divide region G into I demand zones in the discrete set $\mathcal{I} = \{1, 2, ..., I\}$. In particular, we treat EV user demand in each zone $i \in \mathcal{I}$ as a bundle, i.e. one EV charging flow generated from the center of zone i. Assume the charging flow follows Poisson distribution $\pi(\lambda_i^{\max})$, where $\lambda_i^{\max}$ is the maximum EV charging demand rate in zone i.

To better cater to such demand in service region G, the provider first determines J candidate charging station locations in discrete set $\mathcal{J} = \{1, 2, ..., J\}$. Let matrix d_{ij} denote the shortest path distance between the center of zone $i \in \mathcal{I}$ and candidate charging station $j \in \mathcal{J}$. To capture station placement strategy, we introduce a binary variable x_j, where $x_j = 1$ represents that candidate location j is selected for building station and $x_j = 0$ otherwise. The station placement strategy can be further given by the vector $\mathbf{X} = [x_1, x_2, ..., x_J]$. Accordingly, the set of selected charging stations can be characterized by $\mathcal{L} = \{j | x_j = 1, j \in \mathcal{J}\}$ with $L = |\mathcal{L}|$. Suppose each charging station j is equipped with r_j charging spots. In view of the dynamics of system condition and charging demand, we are inspired to characterize the charging process of each station as $M/M/r$ queueing system [15]–[16]. The overall capacity of the system can be given by the vector $\mathbf{R} = [r_1, r_2, ..., r_J]$.

Figure 1 illustrates the typical fast charging system, where service region G is divided into six demand zones. The provider selects three candidate locations to build charging stations. The EV user demand in each zone i generates from the center of zone i and gets charged at nearby stations. Each charging station is modeled as $M/M/r$ queueing system, where r is the number of charging spots.

B. Charging Demand Model

1) Distance preference: We first consider the elastic demand with respect to the driving distance to station. Denote $\Theta(d_{ij}) \in [0, 1]$ as preference of EV charging demand to distance. Following the location science [17]–[18], we define distance preference function as monotone decreasing function of distance to the station, i.e.,

$$\Theta(d_{ij}) = \frac{1}{(1 + d_{ij})^{\alpha_i}}, \alpha_i > 0, \tag{1}$$

where α_i is the distance preference level of EV users in zone i. Without loss of generality, we suppose all users share the same distance preference level, i.e., $\alpha_i = \alpha$ for $i \in \mathcal{I}$.

2) Assignment strategy: We introduce a binary variable $S(i, j)$, where $S(i, j) = 1$ represents that EV users in zone i are assigned to charging station j and $S(i, j) = 0$ otherwise. It should be mentioned that the binary variable $S(i, j)$ determines indivisible charging demand in a zone, i.e. EV users in zone i must be assigned to the same station. This assumption has been widely adopted in existing researches [19]–[20]. Although we only show the case for indivisible charging demand, our approach can also apply to divisible demand case by dividing a zone into smaller zones.

Inspired by [19], we assume that the EV users in zone i are always assigned to the nearest charging station $j \in \mathcal{L}$, i.e.,

$$S(i, j) = \begin{cases} 1, & j = \min_{j \in L} d_{ij} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The arrival demand rate from other zones to station j can be characterized as

$$\Lambda_j^{\max} = \sum_{i \in \mathcal{I}} \lambda_i^{\max} S(i, j) \Theta(d_{ij}), j \in \mathcal{J}. \tag{3}$$

3) Waiting time preference: For any fast charging station, the waiting time is another important factor that affects EV users charging willingness for it. It's obvious that serious congestion will push users away from the station.

Denote W_j as the except waiting time in station j and $\Psi(W_j) \in [0, 1]$ as the waiting time preference function. Intuitively, $\Psi(W_j)$ is monotonically decreasing with waiting time. Based on the above analysis, we are inspired to model waiting time preference function as

$$\Psi(W_j) = \frac{1}{1+\beta W_j}, \beta > 0, j \in \mathcal{L}, \quad (4)$$

where β represents the waiting time preference level.

Under waiting time function, the serving demand rate in station j can be described as

$$\Lambda_j = \Lambda_j^{\max}\Psi(W_j), j \in \mathcal{L} \quad (5)$$

4) $M/M/r$ queueing system: We consider $M/M/r$ queueing system for each fast charging station, where each charging spot has an exponential serving rate μ_j. We suppose the charging spot serving rates in all stations are identical. Without loss of generality, we use μ instead of μ_j. Denote $\rho_j = \Lambda_j/\mu r_j$ as the utilization rate with $\rho_j < 1$. According to [20], the probability that all charging spots are busy is denoted by

$$P_Q(\Lambda_j, r_j) = \frac{(r_j p_j)^{r_j}}{(1-\rho_j)r_j!}\left(\frac{(r_j\rho_j)^{r_j}}{(1-\rho_j)r_j!} + \sum_{k=1}^{r_j-1}\frac{(r_j\rho_j)^k}{k!}\right)^{-1}, \quad (6)$$

On the basis of this, we obtain the average queueing time in station j, i.e.,

$$T_{queue}(\Lambda_j, r_j) = \frac{P_Q(\Lambda_j, r_j)}{\Lambda_j(1-\rho_j)}\rho_j, j \in \mathcal{L}. \quad (7)$$

On the other hand, the average charging time can be characterized as

$$T_{charge} = \frac{1}{\mu}. \quad (8)$$

The excepted waiting time can thus be calculated as

$$W_j = \omega(\Lambda_j, r_j) = T_{charge} + T_{queue}(\Lambda_j, r_j), j \in \mathcal{L}. \quad (9)$$

For any charging station $j \in \mathcal{L}$, substituting Eqn. (9) into Eqn. (5), we have

$$\Lambda_j = \Lambda_j^{\max}\Omega(\Lambda_j, r_j), j \in \mathcal{L}. \quad (10)$$

Eqn.(10) is, in essence, a fixed-point equation . For any charging station, long waiting time will keep potential EV users away from it, which in turn will reduce its waiting time in the future due to the decreased arrival rate.

5) Waiting time constraint: To guarantee the quality of charging services, the expected waiting time satisfies

$$W_j \le \phi, j \in \mathcal{L}, \quad (11)$$

where ϕ is the service level determined by the provider.

C. Problem Formulation

1) Revenue: In this paper, we assume that the unit charging service price p is identical for all EV users. Consider any time interval t, the revenue of station j can be described as

$$U_j = t \times p \times B \times (\mathrm{SoC}_d - \mathrm{SoC}_a) \times \Lambda_j, j \in \mathcal{L}. \quad (12)$$

where B is the capacity of EV battery, SoC_a is arrival state of charge and SoC_d is departure state of charge.

2) Cost: Providing charging service for EV users will inevitably incur operation cost C_{spot}, which is proportional to the capacity of charging station r_j. In addition, there also exists the fixed cost denoted by C_j^b, mainly including necessary energy cost, infrastructure cost, etc. Thus, the overall cost of station j can be described as

$$C_j = x_j(r_j C_{spot} + C_j^b), j \in \mathcal{L}. \quad (13)$$

3) Profit: The profit of charging station j can be defined as the difference between revenue and cost, i.e.,

$$f_j = U_j - C_j, j \in \mathcal{L}. \quad (14)$$

4) FCS placement with elastic demand problem: This paper aims to determine the optimal location and capacity of charging stations to maximize the overall system profit.

$$\begin{aligned}
\textbf{P1}: \quad & \underset{x_j, r_j}{\textbf{Max}} \quad f_{total} = \sum_{j\in\mathcal{L}} f_j && (15)\\
\textbf{s.t.} \quad & \text{Eqn.}(2), && (16)\\
& x_j \in \{0,1\}, \forall j \in \mathcal{J}, && (17)\\
& r_j \ge \frac{\Lambda_j}{\mu}x_j, \forall j \in \mathcal{J}, && (18)\\
& r_j \le r_j^{\max}x_j, \text{integer}, \forall j \in \mathcal{J}, && (19)\\
& \Lambda_j = \Lambda_j^{\max}\Omega(\Lambda_j, r_j), \forall j \in \mathcal{L}, && (20)\\
& W_j \le \phi, \forall j \in \mathcal{L}. && (21)
\end{aligned}$$

Constraint (16) describes the assignment strategy. Constraint (18) ensures the stability of the charging queue, where $r_{\max}$ also reflects the grid security requirements. Constraint (20) captures the fixed-point equation with respect to the serving demand rate Λ_j. Constraint (21) specifies the maximum waiting time requirements.

III. Optimal Locations and capacities for FCS

In this section, we further formulate P1 as a bi-level problem, where the upper level copes with placement issue of charging stations and the lower level determines the optimal capacity of each charging station.

A. Optimal Capacity of Each Fast Charging Station

As for the lower-level problem, we first analyze the sufficient condition for the existence of optimal solution. After that, we propose a searching algorithm based on Newton-Raphson method to find optimal station capacity.

We assume that the provider has determined the optimal location of fast charging stations, i.e. assignment strategy $S(i,j)$ is fixed. Recall the arrival demand rate $\Lambda_j^{\max}$, which is given as well. In this case, the charging demand loss rate for each station is only affected by its congestion state. The optimal capacity for each station $j \in \mathcal{L}$ can thus be determined by solving the following problem

$$\begin{aligned}
\textbf{P2}: \quad & \underset{r_j}{\textbf{Max}} \quad f_j && (22)\\
\textbf{s.t.} \quad & r_j \ge \frac{\Lambda_j}{\mu}, && (23)\\
& r_j \le r_j^{\max}, \text{integer}, && (24)\\
& \Lambda_j = \Lambda_j^{\max}\Omega(\Lambda_j, r_j), && (25)\\
& W_j \le \phi. && (26)
\end{aligned}$$

Theorem 1. *For any r_j, $\Lambda_j^{\max} \ge 0$, the fixed-point equation $\Lambda_j = \Lambda_j^{\max}\Omega(\Lambda_j, r_j)$ has and only has one solution Λ_j when $\Lambda_j \in [0, r_j\mu]$. This solution corresponds to the supply-demand equilibrium.*

Proof. According to Eqn. (6), we have

$$\begin{aligned}\frac{1}{P_Q(\Lambda_j, r_j)} &= 1 + (1-\rho_j)\sum_{k=0}^{r_j-1}(r_j\rho_j)^{k-r_j}\frac{r_j'!}{k!}\\ &= 1 + \sum_{t=1}^{r_j}(1-\rho_j)(\rho_j)^{-t}\frac{r_j!}{(r_j-t)!(r_j)^t},\end{aligned} \tag{27}$$

When $\rho_j \in [0,1]$, and $P_Q(\Lambda_j, r_j)$ monotonously increases with ρ_j. Recall that $\rho_j = \Lambda_j/\mu r_j$, where μ and r_j are constants. Hence we can regard $P_Q(\Lambda_j, r_j)$ as $P_Q(\Lambda_j)$, which monotonously increases with Λ_j.

We rewrite Eqn. (9) as

$$W(\Lambda_j) = \frac{1}{\mu} + \frac{1}{r_j\mu - \Lambda_j}P_Q(\Lambda_j, \mu, r_j). \tag{28}$$

Since $1/(r_j\mu - \Lambda_j)$ monotonously increases with Λ_j, $W(\Lambda_j)$ increases with Λ_j. Taking into account the fact that congestion preference function $\Psi(W_j)$ increasing with waiting time, $\Omega(\Lambda_j, r_j)$ is decreasing with Λ_j.

According to the fixed-point equation Eqn. (25), let $F(\Lambda_j) = \Lambda_j - \Lambda_j^{\max}\Omega(\Lambda_j, r_j)$.

On one hand, we have

$\lim_{\Lambda_j \to 0} F(\Lambda_j) = \lim_{\Lambda_j \to 0} \Lambda_j - \Lambda_j^{\max}\Omega(\Lambda_j, r_j) = -\Lambda_j^{\max} < 0.$

On the other hand, we have

$\lim_{\Lambda_j \to r_j\mu} F(\Lambda_j) = \lim_{\Lambda_j \to r_j\mu} \Lambda_j - \Lambda_j^{\max}\Omega(\Lambda_j, r_j) = r_j\mu > 0.$

Since $F(\Lambda_j)$ increases with Λ_j, $F(\Lambda_j) = 0$ has and only has one root when $\Lambda_j \in [0, r_j\mu]$. □

Theorem 2. *There exists an optimal solution r_j^* to P2 if $\max\{r_j^w, \frac{\Lambda_j}{\mu}\} \le r_j^{\max}$, where $r_j^w = \min\{r_j | \omega(\Lambda_j^{\max}, r_j) \le \phi\}$.*

Proof. Note that for $r_j \ge r_j^w$, the waiting time constraint is satisfied because $\omega(\Lambda_j, r_j) \le \omega(\Lambda_j^{\max}, r_j) \le \phi$. On the other hand, the stability of charging queue is ensured for $r_j \ge \frac{\Lambda_j}{\mu}$. Thus, if the upper bound of capacity $r_j^{\max}$ is greater than $\max\{r_j^w, \frac{\Lambda_j}{\mu}\}$, the feasibility of P2 is guaranteed. □

Based on the above analysis, the fixed-point equation Eqn. (25) can be solved naturally. We adopt Newton-Raphson method [21] to compute the numerical solution of Eqn. (25). In particular, Newton-Raphson method can get accurate result for the fixed-point equation Eqn. (25). It iterates as follows and finally converges when $|\Lambda_j^{k+1} - \Lambda_j^k| < \epsilon$

$$\Lambda_j^{k+1} = \Lambda_j^k - (\frac{dF}{d\Lambda_j})^{-1}F(\Lambda_j^k), \tag{29}$$

where ϵ is small constant. The derivative can be estimated by the neighborhood values, where δ is small constant

$$\Lambda_j^{k+1} = \Lambda_j^k - (\frac{F(\Lambda_j^k + \delta) - F(\Lambda_j^k - \delta)}{2\delta})^{-1}F(\Lambda_j^k + \delta). \tag{30}$$

Given any station capacity r_j, we can obtain the profit f_j and serving demand rate Λ_j according to Eqn. (30). Actually, the maximal profit f_j^* and corresponding station capacity r_j^* are optimal solutions to P2.

From Algorithm 1, we can easily observe that for any station j, the maximal profit f_j^* and optimal capacity r_j^* depend on the arrival demand rate $\Lambda_j^{\max}$. Thus, we build two searching tables with respect to $(\Lambda_j^{\max}, f_j^*)$ and $(\Lambda_j^{\max}, r_j^*)$ during the preprocessing stage. After that, the optimal capacity problem P2 can be solved via a simple table lookup.

Algorithm 1: Charging Station Optimal Capacity Algorithm

Input: $r_j^{\max}$,ϕ_j, $\lambda_i^{\max}$, μ, t, p, B, Sod$_d$, Sod$_a$, C_{spot}, C_j^b
Output: f_j^*, r_j^*, Λ_j^*

```
for r_j ∈ [Λ_j/μ, r_max] do
    Determine the station j's serving demand rate Λ_j by
    solving Eqn. (25) and compute waiting time W_j;
    if W_j ≤ φ then
        Determine profit of station j;
        Update the maximal profit f_j^*, Λ_j^* and r_j^*;
```

B. Optimal Locations of Fast Charging Stations

In this part, we analyze the upper level problem of P1. Fig. 2 illustrates the main challenge of the upper level problem. To serve EV users' charging demand from 4 demand zones ($Z1, Z2, Z3, Z4$) with different maximum charging demand rates $\lambda_i^{\max}(i = 1,2,3,4)$, the provider selects two candidate locations (S2, S3) to build charging stations. The distances between demand zones and charging stations are depicted in this figure. Each zone will be assigned to one closer station. Taking into account demand loss with respect to driving distance and station congestion, we obtain serving demand rates (Λ_2,Λ_3) and profit of stations (f_2,f_3) by Algorithm 1. The challenge is that we will get different f_{total} if choosing different station locations. In fact, there are 2^J possible scenarios. Obviously, the exhaustive method no longer apply for this problem.

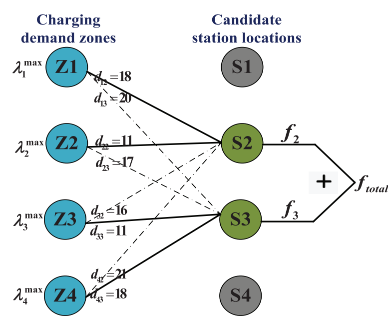

Fig. 2. An Example of Upper Level Problem

To this end, we propose an ascent heuristic algorithm to solve the upper level problem, as shown in Algorithm 2. In order to determine the optimal number of selected charging stations, all possible selected station numbers need to be examined. For each selected station number $L \in [1, J]$, we first choose L station locations from $\mathcal{J}$ as the initial set $\mathcal{L}$ (Lines 2-5), and exchange the remaining elements of $\mathcal{J}$ with one of $\mathcal{L}$ in turn where the new $\mathcal{L}$ that provides the largest profit will be selected. Repeat this process until profit does not change (Lines 6-16).

Algorithm 2: Ascent Heuristic Algorithm

Input: the number of iteration $iter$
Output: location vector $\mathbf{X}$, capacity vector $\mathbf{R}$, profit f_{total}

1 **for** $L \in [1, J]$ **do**
2 Randomly choose L station locations from $\mathcal{J}$ as the initial set $\mathcal{L}$;
3 Unselected charging station set $\mathcal{T} = \mathcal{J} - \mathcal{L}$;
4 Calculate initial profit $f_{total}^{L,0}$ by Algorithm 1;
5 Set $f_{dif}^{L} \leftarrow 1$, $i \leftarrow 0$;
6 **while** $f_{dif}^{L} > 0$ **do**
7 $i \leftarrow i + 1$;
8 Randomly choose $l \in \mathcal{L}$;
9 Exchange l with every $t \in \mathcal{T}$ and calculate the profit, $\mathbf{X}$, $\mathbf{R}$ by Algorithm 1;
10 Find the maximal profit $f_{total}^{L,i}$, $\mathbf{X}$, and $\mathbf{R}$;
11 Update $\mathcal{L}$;
12 **if** $i > iter$ **then**
13 $f_{dif}^{L} \leftarrow f_{total}^{L,i} - f_{total}^{L,i-iter}$;
14 $f_{total}^{L} \leftarrow f_{total}^{L,i}$;
15 Find the maximal profit $f_{total} = \max_{L} f_{total}^{L}$, $\mathbf{X}$ and $\mathbf{R}$;

IV. Simulation results

A. Setup

Divide the service region of interest into 80 demand zones. For each zone i, there exists one stream of Poisson charging demand rate $\lambda_i^{\max}$ randomly generated from the interval $[80, 95]$. Without loss of generality, the candidate charging station location set $\mathcal{J}$ coincides with demand zones set $\mathcal{I}$. To facilitate simulation, the EV type is captured by Nissan Leaf PEV [22] which has 24 kWh battery capacity. Assume that the arriving SoC (SoC_a) is 0 and the departure SoC (SoC_d) is 100%. According to [23], each charging spot in the station can serve three EV users in an hour. We set C_{spot} and C_j^b as \$23500 and \$163000 [22].

To demonstrate the performance of our approach, we compare the proposed algorithm with two baseline methods.

- *Uniform Distributed Capacity Algorithm (UDCA)*: UDCA assigns the same number of charging spots in different charging stations among lower level, while it is similar to proposed algorithm in upper level.
- *Random Location Algorithm (RLA)*: RLA randomly chooses p candidate locations to build stations in the upper level, while it is same as the proposed algorithm in the lower level.

B. Performance

We compare the performance of proposed algorithm and Exhaustive Search Method regarding to the system profit and runtime. We randomly choose J zones (J from 16 to 24) from 80 zones as the candidate charging stations. From Fig. 3, we observe that as system scale increases, the runtime of Exhaustive Search Method increases exponentially. The proposed algorithm can achieve an approximation of optimal profit in a much more efficient way.

Figure 4 presents the system profit achieved in proposed algorithm with varying charging station number. We observe that the profit increases at first and goes down subsequently, because the revenue from constructing a new station can not compensate for the cost. We also illustrate the impact of building cost on profit. The fixed cost C_j^b may be influenced by government subsidies, building materials price or others. When the fixed cost decreases, the total system profit increases. Due to the decrease of fixed cost, the charging service provider is motivated to build more charging stations. Accordingly, the optimal charging station number which can achieve the maximal profit increases as well.

Figure 5 presents the demand loss rate of proposed algorithm with varying charging station number. With the increase of charging station number, the demand loss rate first falls quickly and slows down later due to the marginal effect.

We compare the system profit of the proposed algorithm against baselines in Fig. 6. As the selected charging station number increases, the proposed algorithm outperforms other baselines. Fig. 7 illustrates the comparison in demand loss rate. We observe that larger the selected charging station number leads to lower demand loss rate. The proposed algorithm can achieve lower demand loss rate than RLA. UDCA performs better than the proposed algorithm with the increase of selected charging station number. However, in UDCA, the charging spots in each charging station are under utilized, making the profit of UDCA less than the proposed algorithm.

Figure 8 illustrates the optimal selected charging station number and corresponding profit with varying EV user number (the sum of $\lambda_i^{\max}$ in all zones). As EV user number increases, the profit keeps going up. The proposed algorithm performs better than other baselines.

V. Conclusion

This paper studies the FCS placement problem especially with the elastic charging demand, which is pertaining to distance preference and waiting time preference. We first propose a fixed-point equation to illustrate the relationship between serving demand rate and waiting time at station. A heuristic algorithm is proposed to determine the optimal location and capacity of charging stations. Simulation results demonstrate the effectiveness of our approach in improving system profit and reducing demand loss rate.

VI. Acknowledgement

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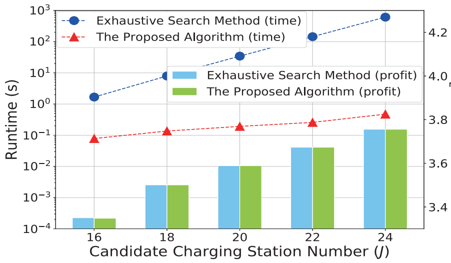

Fig. 3. System profit and runtime with respect to candidate charging station number

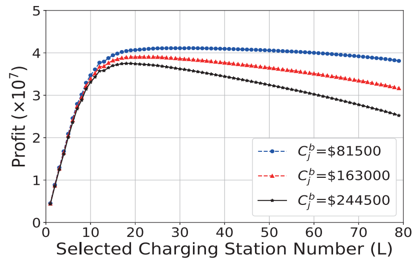

Fig. 4. System profit with respect to selected charging station number

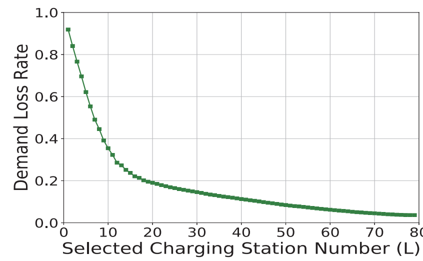

Fig. 5. Demand loss rate with respect to selected charging station number

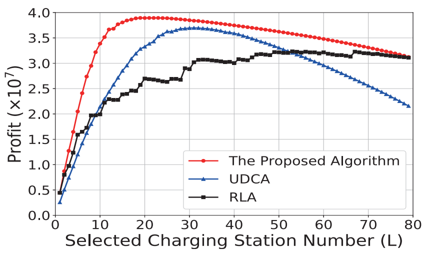

Fig. 6. System profit with respect to selected charging station number

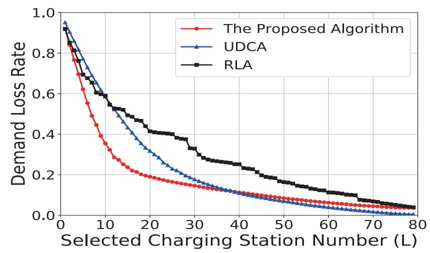

Fig. 7. Demand loss rate with respect to selected charging station number

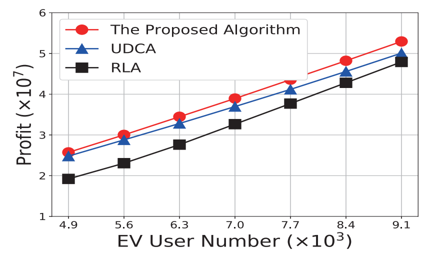

Fig. 8. System profit with respect to EV users number

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