Cooperative Spectrum Sharing in D2D-Enabled Cellular Networks

Chuan Ma, Yuqing Li, Hui Yu, Xiaoying Gan, Xinbing Wang, Yong Ren, and Jun (Jim) Xu

***Abstract*— Device-to-device (D2D) communication underlaying cellular networks is a promising technology for improving network resource utilization, and cooperative communication technology is usually used to mitigate the interference caused by D2D communication. Due to the additional signal processing cost introduced by cooperative communication, the cellular links who have the exclusive usage right of the network spectrum can charge the D2D links a fee for spectrum usage to enhance their profit. In this paper, we propose a contract-based cooperative spectrum sharing mechanism to exploit transmission opportunities for the D2D links and meanwhile achieve the maximum profit of the cellular links. We first design a cooperative relaying scheme that employs superposition coding at both the cellular transmitters and D2D transmitters. The cooperative relaying scheme can maximize the data rate of the D2D links without deteriorating the performance of the cellular links. Then, we employ a contract-theoretic framework to model the spectrum trading process based on the cooperative relaying scheme, and derive the optimal power-payment contracts for the cellular links under both the cases that the private information (i.e., channel quality) of the D2D links is continuous and discrete using tools from continuous- and discrete-time optimal control theories, respectively. Analytic and numerical results confirm the efficiency of the proposed spectrum sharing mechanism.**

***Index Terms*— D2D communication, cellular network, cooperative relaying, superposition coding, contract theory.**

I. Introduction

RECENTLY, the wireless cellular networks have seen a rapid increase in the demand of local area services and proximity services (ProSe) among the highly-capable user equipments (UEs). In this context, a new technology called device-to-device (D2D) communication, which enables direct communication between UEs that are in proximity, has been proposed and has strongly appealed to both academia [2]–[5] and industry [6], [7]. The adoption of D2D communication in cellular networks holds the promise of many types of gains [8], such as allowing for high-rate low-delay transmission for proximity services, tightening the frequency reuse factor, extending the cellular coverage and facilitating new types of peer-to-peer services.

The adoption of D2D communication in cellular networks also brings a number of technical challenges, for example device and service discovery, mode selection and intra-cell interference mitigation. Specifically, intra-cell interference becomes a major issue in D2D-enabled cellular networks, since the D2D links share the spectrum with the regular cellular links and the interference between these two types of links would severely hamper the performance of the network. To guarantee a reliable communication in the network, effective intra-cell interference mitigation schemes should be designed. Many works have been done on this topic [9]–[13]. Min *et al*. [9] proposed an interference cancellation scheme that exploits a re-transmission of the interference from the base station. Kaufman *et al*. [10] developed a discovery protocol that establishes a route to connect the D2D users to the intended destination with minimal interference. Zihan *et al*. [11] proposed a collision avoidance mechanism that creates an exclusion region around the D2D receiver to prohibit interferers from transmitting a signal. Yang *et al*. [12] designed an interference alignment scheme that controls the peak interference power on each interfered cellular link under a certain threshold. Song *et al*. [13] presented a joint power and rate control scheme for the D2D-enabled cellular network with successive interference cancellation capabilities to control the mutual interference.

Cooperative communication technology [14], [15], which allows multiple nodes to relay each other's transmissions, is also a promising technology for mitigating the mutual interference in the network. Best known cooperative relaying protocols include Amplify-and-Forward (AF) protocol, Decode-and-Forward (DF) protocol and Coded Cooperation (CC) protocol [16]. Superposition coding (SC) [17] has also been applied in DF protocol to further improve the cooperation gain [18]–[22]. Popovski and De Carvalho. [18] introduced a two-step relaying scheme where the source broadcasts the message using SC. Han *et al.* [19] and Shin and Kim [20] presented relaying protocols for cognitive radio networks, where the secondary transmitter employs SC-based DF relaying to transmit the primary signal along with the secondary signal, such that the outage performance of the primary system is not affected. Wui and Kim [21] designed a relaying scheme for the relay-multicasting system where the relay decodes and forwards multicast messages from the source to multiple destinations that multiplexes the unicast and multicast messages at the source using two-level SC. Recently, SC has also been applied for D2D communication in

Manuscript received January 14, 2016; revised April 30 2016 and July 4, 2016; accepted August 7, 2016. Date of publication August 16, 2016; date of current version October 14, 2016. This work is supported by the NSF China (Nos.61532012, 61325012, 61271219, 61521062, 61428205, 91438115) and the US NSF grant NeTS. The associate editor coordinating the review of this paper and approving it for publication was M. Abdallah.

C. Ma, Y. Li, H. Yu, X. Gan, and X. Wang are with the Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai 200240, China (e-mail: oknewkimi@sjtu.edu.cn; liyuqing@sjtu.edu.cn; yuhui@sjtu.edu.cn; ganxiaoying@sjtu.edu.cn; xwang8@sjtu.edu.cn).

Y. Ren is with the Department of Electronic Engineering, Tsinghua University, Beijing 100084, China (e-mail: reny@tsinghua.edu.cn).

J. Xu is with the College of Computing, Georgia Institute of Technology, Atlanta, GA 30332 USA (e-mail: jx@cc.gatech.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TCOMM.2016.2600674

cellular networks. In [22], Shalmashi *et al.* proposed a cooperative communication scheme that allows the D2D transmitter to transit the cellular data and its own data by employing SC within equal-length phases. In the above works, SC is used at only one transmitter. However, in this paper we design a cooperative relaying scheme that employs SC at both the cellular transmitter and D2D transmitter, which can further improve the spectrum efficiency of the network.

On the other hand, due to the additional signal processing cost by using cooperative communication technology, the cellular links who have the exclusive usage right of the network spectrum can charge the D2D links a fee for spectrum usage to enhance their profit. However, in the presence of asymmetric information between the cellular links and D2D links (i.e., the cellular links have no local information such as channel quality of the D2D links), how to design effective spectrum trading mechanisms is challenging. Some mechanisms have been employed to tackle the spectrum trading issue, for example pricing mechanisms [23], [24], auction mechanisms [25], [26], and gaming mechanisms [27], [28]. The pricing mechanism is designed mainly for the *complete* information scenario, and thereby does not apply to the *incomplete* information scenario studied in this paper. The auction mechanism requires *multiple* buyers (bidders), and thereby is not the right framework for the *single* buyer (D2D link) spectrum trading problem studied in this paper. Contract theory [29], which deals with the incentive compatible problem in a monopoly market with incomplete information, is appropriate for model such a scenario. Contract theory has been applied for network management in cognitive network [30]–[32], relaying network [33], [34], heterogeneous network [35], [36], etc. Gao *et al.* [30], [31] designed monopolist-dominated quality-price contracts for spectrum trading between a single primary user and multiple secondary users. Kalathil and Jain [32] studied the incentive mechanism for spectrum sharing in cognitive networks that achieves socially optimal rate allocations with contracts in licensed bands. Duan *et al.* [33] and Hasan and Bhargava [34] proposed cooperative relaying schemes through contracts with access-time incentive and monetary incentive respectively. Peng *et al.* [35] and Li *et al.* [36] applied contract theory to the interference management and data offloading problems in heterogeneous networks respectively. Recently, contract theory has also been applied for D2D communication. Zhang *et al.* [37] classified the users' preferences toward D2D communication into a set of types, and designed a contract-theoretic framework to incentivize the users to engage in D2D communication. Different from [37], in this paper we focus on the cooperative relaying scenario, and employ contract mechanism to incentivize the D2D links to engage in the cooperative relaying communication and meanwhile maximize the profit of the cellular links.

In this paper, we propose a contract-based cooperative spectrum sharing mechanism to exploit transmission opportunities for the D2D links and meanwhile maximize the profit of the cellular links in D2D-enabled cellular networks. The main contributions of this paper are summarized as follows.

- We design a superposition coding-based cooperative relaying scheme for D2D communication underlaying a cellular (multicasting) system, which can maximize the data rate of the D2D link without degrading the performance of the cellular system. Different from traditional cooperative relaying schemes that employ superposition coding at only one type of transmitter, the proposed scheme employs superposition coding at two types of transmitters (i.e., both the cellular transmitter and D2D transmitter), and thereby can achieve a higher network performance in terms of D2D data rate.
- We model the cooperative relaying-based spectrum trading process between the cellular system and D2D link by a principal-agent framework. Under this framework, the cellular system acts as a principal and offers a power-payment contract to the D2D link, and the D2D link acts as an agent and chooses the contract item that maximizes its utility. We derive the optimal contracts for both the cases that the private information of the D2D link is continuous and discrete, using tools from continuous- and discrete-time optimal control theories respectively. The proposed mechanism can incentivize the D2D link to engage in the cooperative relaying communication and meanwhile maximize the profit of the cellular system.

The remaining part of this paper is organized as follows. Section II presents system model. Section III proposes the cooperative relaying scheme. Sections IV and V study the continuous and discrete contracts respectively. Section VI presents numerical results and Section VII concludes the paper.

II. System Model

A. Network Model

We consider a hybrid network consisting of multiple cellular (multicasting) systems and multiple D2D links. We assume that each cellular system can cooperate with one D2D link by using a two-layer superposition coding scheme. Thus, without loss of generality, in this paper we focus on one cellular system and one D2D link. In the cellular system, a cellular transmitter (CT) multicasts data to K cellular receivers (CRs) with transmission power P_C. Over the D2D link, a D2D transmitter (DT) transmits data to a D2D receiver (DR) with transmission power P_D. Let x_M and x_D respectively denote the cellular multicasting signal and D2D signal, with $E\{x_M\} = E\{x_D\} = 0$ and $E\{|x_M|^2\} = E\{|x_D|^2\} = 1$. We denote the channel coefficients of CT-CR$_i$, CT-DT, CT-DR, DT-DR, DT-CR$_i$ links by h_{C_i}, h_{CDT}, h_{CDR}, h_D, h_{DC_i} respectively, and assume the background noise for each link to be independent Gaussian, i.e., $n \sim \mathcal{CN}\left(0, \sigma^2\right)$. We use $\gamma_j = \frac{|h_j|^2}{\sigma^2}$ to denote the channel quality of link $j \in \{C_i, CDT, CDR, D, DC_i\}$, and express the achievable data rate of a link with signal-to-interference-plus-noise ratio (SINR) x by $C(x) = \log(1 + x)$.

B. Spectrum Sharing Model

Suppose the cellular system has the exclusive usage right of the network spectrum, and the D2D link cannot access the spectrum without the permission from the cellular system. For the cellular system, sharing spectrum with the D2D link brings in extra interference. Therefore, to cancel the interference from the D2D link and thus guarantee the performance of the cellular system, in this paper we employ a cooperative relaying scheme for the cellular system and D2D link. The cooperative relaying protocol consists of two phases. In the first phase, CT broadcasts the cellular signal; DT receives and decodes the cellular signal. In the second phase, DT regenerates the cellular signal, superposes it with the D2D signal, and then broadcasts the composite signal; CRs and DR decode the interested signals from the received composite signal respectively. In Section III, we will study the optimal time and power allocation problem for this scheme.

On the other hand, considering the cooperative relaying scheme will introduce additional signal processing cost to the cellular system, we assume the cellular system can charge the D2D link a fee to enhance its profit. Denote the cost of the cellular system for cooperating with the D2D link (whose transmission power is P_D)[1] by $\pi(P_D)$, and suppose the D2D link pays the cellular system a fee $T(P_D)$ for spectrum usage. Then, the utility of the cellular system can be defined as

$$U_C = T(P_D) - \pi(P_D), \tag{1}$$

and the utility of the D2D link can be defined as

$$U_D = R_D(P_D, \gamma_D) - \mu T(P_D), \tag{2}$$

where $R_D(P_D, \gamma_D)$ denotes the achievable data rate of the D2D link by employing the cooperative relaying scheme, and μ is the weight for the payment.

We further assume that the cellular system has no exact knowledge of the D2D channel quality γ_D, which is the private information of the D2D link, but only some statistical information about γ_D, e.g., the probability density function $f(\gamma_D)$. In this case, the principal-agent model is suitable for modeling the spectrum trading process between the cellular system and D2D link. The cellular system acts as a principal and assigns a transmission power $P_D(\gamma_D)$ and a payment $T(P_D(\gamma_D))$ for each $\gamma_D \in \Upsilon$ (Υ is the set of all possible γ_D) to maximize its expected utility $\mathbb{E}(U_C)$. We refer to such set of power-payment bundles as a contract $\Phi = \{(P_D(\gamma_D), T(P_D(\gamma_D))), \forall \gamma_D \in \Upsilon\}$. The cellular system offers Φ to the D2D link, and the D2D link acts as an agent and chooses the contract item (i.e., power-payment bundle) that maximizes its utility. Thus, the **o**ptimal **c**ontract **d**esign problem for the cellular system can be formulated as

Problem OCD:

$$\max_{T(\cdot)} \mathbb{E}\left[T\left(\hat{P}_D(\gamma_D)\right) - \pi\left(\hat{P}_D(\gamma_D)\right)\right] \tag{3}$$

$$\text{s.t. } \hat{P}_D(\gamma_D) = \arg\max_{P_D(\cdot)} R_D(P_D(\gamma_D), \gamma_D) - \mu T(P_D(\gamma_D)). \tag{4}$$

In Sections IV and V, we will derive the optimal solutions of Problem OCD for the cases that γ_D is continuous and discrete respectively.

According to the above model, the interaction between the cellular system and D2D link in the contract-based cooperative spectrum sharing mechanism involves the following steps, as shown in Fig. 1:

- First, the cellular system calculates the values of $\pi(P_D)$ and $R_D(P_D(\gamma_D), \gamma_D)$ according to the cooperative relaying scheme, and then obtains the optimal contract Φ by solving Problem OCD and offers it to the D2D link;
- Then, the D2D link chooses the optimal contract item (including transmission power P_D and payment T) and informs the cellular system of its choice;
- After receiving the information from the D2D link, the cellular system sends the D2D link a CRS request to trigger the cooperative relaying procedures;
- After receiving the CRS request, the D2D link executes the cooperative relaying scheme with transmission power P_D and pays the cellular system a fee T.

[1]Note that according to the contract model formulated in the following part, P_D is a function of γ_D. Thus, $\pi(P_D)$ and $T(P_D)$ are in fact short for $\pi(P_D(\gamma_D))$ and $T(P_D(\gamma_D))$ respectively.

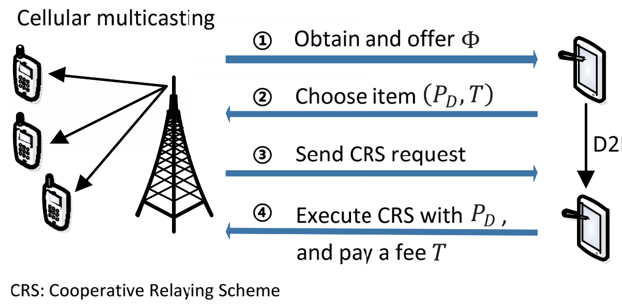


Fig. 1. Interaction between the cellular system and D2D link.

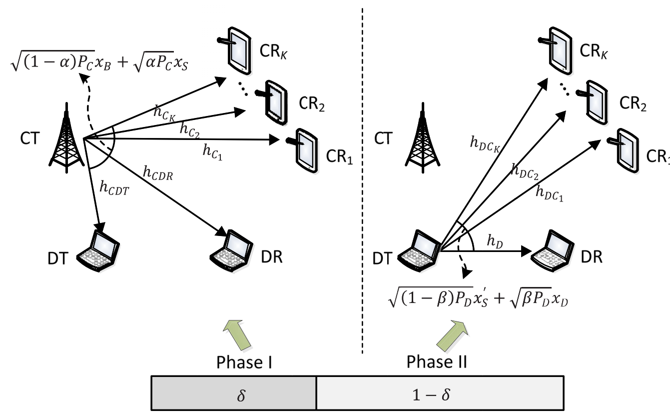


Fig. 2. Two-phase cooperative relaying scheme.

III. Cooperative Relaying Scheme

In this section, we describe the cooperative relaying scheme and study the corresponding parameter design problem.

A. Scheme Description

We consider a two-phase transmission scheme in a unit-time slot, where the first phase takes time δ $(0 \le \delta \le 1)$ and the second phase takes time $1-\delta$, as shown in Fig.2. The transmission scheme is described as follows:

Phase 1: CT splits its signal into two parts, say the basic signal x_B and the superposed signal x_S, with $E\{x_B\} = E\{x_S\} = 0$ and $E\{|x_B|^2\} = E\{|x_S|^2\} = 1$, and broadcasts these two parts in a composite signal $\sqrt{(1-\alpha)P_C}x_B + \sqrt{\alpha P_C}x_S$, where

α $(0 \le \alpha \le 1)$ is the power allocation factor at CT. Denoting the signals received by CR$_i$, DT, DR by $y_{CR_i}^{(1)}$, $y_{DT}^{(1)}$, $y_{DR}^{(1)}$ respectively, we have

$$y_{CR_i}^{(1)} = h_{C_i}\left(\sqrt{(1-\alpha)P_C}x_B + \sqrt{\alpha P_C}x_S\right) + n_{CR_i}, \quad (5)$$

$$y_{DT}^{(1)} = h_{CDT}\left(\sqrt{(1-\alpha)P_C}x_B + \sqrt{\alpha P_C}x_S\right) + n_{DT}, \quad (6)$$

$$y_{DR}^{(1)} = h_{CDR}\left(\sqrt{(1-\alpha)P_C}x_B + \sqrt{\alpha P_C}x_S\right) + n_{DR}, \quad (7)$$

where the superscript (1) denotes the first phase and n_{CR_i}, n_{DT}, n_{DR} denote the noise at CR$_i$, DT, DR respectively. Then DT, DR successively decode x_B and x_S. CR$_i$ does not decode the signals, but keeps $y_{CR_i}^{(1)}$ in memory.

Phase 2: DT first generates the signal x_S', which contains identical information as x_S decoded in phase 1 but is re-encoded with a different codebook, and then linearly combines the signal x_S' with power $(1-\beta)P_D$ and its own signal x_D with power βP_D to generate a composite signal $z_D = \sqrt{(1-\beta)P_D}x_S' + \sqrt{\beta P_D}x_D$, where β $(0 \le \beta \le 1)$ is the power allocation factor at DT. Then DT broadcasts z_D and CR$_i$, DR receive $y_{CR_i}^{(2)}$, $y_{DR}^{(2)}$ respectively,

$$y_{CR_i}^{(2)} = h_{DC_i}\left(\sqrt{(1-\beta)P_D}x_S' + \sqrt{\beta P_D}x_D\right) + n_{CR_i}, \quad (8)$$

$$y_{DR}^{(2)} = h_D\left(\sqrt{(1-\beta)P_D}x_S' + \sqrt{\beta P_D}x_D\right) + n_{DR}, \quad (9)$$

where the superscript (2) denotes the second phase. CR$_i$ decodes x_S', and thereby x_S, from $y_{CR_i}^{(2)}$ by treating the interfering signal x_D as noise. Then CR$_i$ cancels x_S from $y_{CR_i}^{(1)}$ and obtains the signal

$$\begin{aligned} y_{CR_i}^{(1)-} &= y_{CR_i}^{(1)} - h_{C_i}\sqrt{\alpha P_C}x_S \\ &= h_{C_i}\sqrt{(1-\alpha)P_C}x_B + n_{CR_i}, \end{aligned} \quad (10)$$

from which x_B can be decoded in the absence of interference. On the other hand, DR first generates x_S' from x_S which is locally decoded in phase 1, and then cancels it from $y_{DR}^{(2)}$ and obtains the signal

$$\begin{aligned} y_{DR}^{(2)-} &= y_{DR}^{(2)} - h_D\sqrt{(1-\beta)P_D}x_S' \\ &= h_D\sqrt{\beta P_D}x_D + n_{DR}. \end{aligned} \quad (11)$$

Then x_D can be decoded free of interference.

B. Performance Analysis

First, we study the performance of the cellular system. Denoting the data rates of x_B and x_S over link CT-DT in phase 1 by $R_{B-DT}^{(1)}$, $R_{S-DT}^{(1)}$ respectively, we have

$$R_{B-DT}^{(1)} = \delta C\left(\frac{(1-\alpha)\gamma_{CDT}P_C}{1+\alpha\gamma_{CDT}P_C}\right), \quad (12)$$

$$R_{S-DT}^{(1)} = \delta C(\alpha\gamma_{CDT}P_C). \quad (13)$$

Denoting the data rates of x_B and x_S over link CT-DR in phase 1 by $R_{B-DR}^{(1)}$, $R_{S-DR}^{(1)}$ respectively, we have

$$R_{B-DR}^{(1)} = \delta C\left(\frac{(1-\alpha)\gamma_{CDR}P_C}{1+\alpha\gamma_{CDR}P_C}\right), \quad (14)$$

$$R_{S-DR}^{(1)} = \delta C(\alpha\gamma_{CDR}P_C). \quad (15)$$

Thus, the data rates of x_B and x_S in phase 1 are

$$\begin{aligned} R_B^{(1)} &= \min\left\{R_{B-DT}^{(1)}, R_{B-DR}^{(1)}\right\} \\ &= \delta C\left(\frac{(1-\alpha)\gamma_{CD\min}P_C}{1+\alpha\gamma_{CD\min}P_C}\right), \end{aligned} \quad (16)$$

$$R_S^{(1)} = \min\left\{R_{S-DT}^{(1)}, R_{S-DR}^{(1)}\right\} = \delta C(\alpha\gamma_{CD\min}P_C), \quad (17)$$

where $\gamma_{CD\min} = \min\{\gamma_{CDT}, \gamma_{CDR}\}$. In phase 2, CR$_i$ successively decodes x_S from $y_{CR_i}^{(2)}$ and x_B from $y_{CR_i}^{(1)-}$. Denoting the corresponding data rates by $R_{S-CR_i}^{(2)}$ and $R_{B-CR_i}^{(2)}$, we have

$$R_{S-CR_i}^{(2)} = (1-\delta)C\left(\frac{(1-\beta)\gamma_{DC_i}P_D}{1+\beta\gamma_{DC_i}P_D}\right), \quad (18)$$

$$R_{B-CR_i}^{(2)} = \delta C((1-\alpha)\gamma_{C_i}P_C). \quad (19)$$

Thus, the data rates of x_S and x_B in phase 2 are

$$\begin{aligned} R_S^{(2)} &= \min_i\left\{R_{S-CR_i}^{(2)}\right\} \\ &= (1-\delta)C\left(\frac{(1-\beta)\gamma_{DC\min}P_D}{1+\beta\gamma_{DC\min}P_D}\right), \end{aligned} \quad (20)$$

$$R_B^{(2)} = \min_i\left\{R_{B-CR_i}^{(2)}\right\} = \delta C((1-\alpha)\gamma_{C\min}P_C), \quad (21)$$

where $\gamma_{DC\min} = \min_i\left\{\gamma_{DC_i}\right\}$ and $\gamma_{C\min} = \min_i\left\{\gamma_{C_i}\right\}$. Therefore, the achievable cellular data rate can be given by

$$R_C = \min\left\{R_B^{(1)}, R_B^{(2)}\right\} + \min\left\{R_S^{(1)}, R_S^{(2)}\right\}. \quad (22)$$

Next, we study the performance of the D2D link. Denote the achievable D2D data rate by $R_D(P_D, \gamma_D)$, then we have

$$R_D(P_D, \gamma_D) = (1-\delta)C(\beta\gamma_D P_D). \quad (23)$$

C. Optimal Parameter Design

Let $R_C^{(0)}$ denote the achievable data rate of the cellular system for the non-spectrum sharing case where the D2D link is not allowed to access the network, then

$$R_C^{(0)} = \min_i\left\{C(\gamma_{C_i}P_C)\right\} = C(\gamma_{C\min}P_C). \quad (24)$$

To prioritize the transmission of the cellular system, the cooperative relaying scheme should satisfy $R_C \ge R_C^{(0)}$. Therefore, we proceed towards the following parameter design problem for the **c**ooperative **r**elaying **s**cheme, which aims at maximizing the D2D data rate while guaranteeing the performance of the cellular system:

Problem CRS:

$$\max_{0\le\alpha,\beta,\delta\le 1} R_D(P_D, \gamma_D) \quad (25)$$

$$\text{s.t. } R_C \ge R_C^{(0)}. \quad (26)$$

In order to find the optimal α, β, δ, i.e., α^*, β^*, δ^*, we first propose the following two lemmas.

Lemma 1: At the optimal solution of Problem CRS, the following condition holds: $R_B^{(1)} = R_B^{(2)}$ *and* $R_S^{(1)} = R_S^{(2)}$.

Proof: First, at DT, the transmit rate of x_S in phase 2 shall not exceed the receive rate of x_S in phase 1, i.e., $R_S^{(1)} \ge R_S^{(2)}$. Furthermore, if $R_S^{(1)} > R_S^{(2)}$, we can increase the D2D data rate while not affecting the cellular data rate by decreasing δ

and increasing β. Therefore, we have $R_S^{(1)} = R_S^{(2)}$. Similarly, we have $R_B^{(1)} = R_B^{(2)}$. ■

Lemma 2: At the optimal solution of Problem CRS, the following condition holds: $R_C = R_C^{(0)}$.

Proof: By Lemma 1, the constraint (26) is equivalent to $R_B^{(2)} + R_S^{(2)} \geq R_C^{(0)}$. Given δ, if $R_B^{(2)} + R_S^{(2)} > R_C^{(0)}$, we can increase the D2D data rate by increasing β until $R_B^{(2)} + R_S^{(2)} = R_C^{(0)}$. Thus, we have $R_C = R_C^{(0)}$ at the optimal solution. ■

By Lemmas 1 and 2, we have the following proposition.

Proposition 1: The optimal solution of Problem CRS is

$$\begin{cases} \delta^* = \dfrac{C\left(\gamma_{Cmin} P_C\right)}{C\left(\gamma_{CDmin} P_C\right)}, & (27)\\ \alpha^* = \min\left\{\dfrac{1}{P_C}\left(\frac{1}{\gamma_{Cmin}} - \frac{1}{\gamma_{CDmin}}\right), 1\right\}, & (28)\\ \beta^* = \dfrac{1}{b} + \left(\frac{1}{b} - 1\right)\frac{1}{\gamma_{DCmin} P_D}, & (29) \end{cases}$$

where $b = \left(1 + \alpha^* \gamma_{CDmin} P_C\right)^{\frac{\delta^*}{1-\delta^*}}$.

Proof: From $R_B^{(1)} = R_B^{(2)}$, we obtain (28). Then from $R_S^{(1)} = R_C^{(0)} - R_B^{(1)}$ we obtain (27), and from $R_S^{(1)} = R_S^{(2)}$ we obtain (29). ■

Then, we can obtain the maximum achievable data rate of the D2D link as

$$R_D^*\left(P_D, \gamma_D\right) = \left(1 - \delta^*\right) C\left(\beta^* \gamma_D P_D\right), \tag{30}$$

where β^* and δ^* are given in Proposition 1.

Remark 1. In consideration of the feasibility of Problem CRS, the optimal solution should satisfy $\delta^* < 1$ and $\alpha^*, \delta^* > 0$. This requires

$$\gamma_{C\min} < \gamma_{CD\min}, \tag{31}$$

and

$$1 + \frac{C\left(\alpha^* \gamma_{CD\min} P_C\right)}{C\left(\gamma_{DC\min} P_D\right)} < \frac{C\left(\gamma_{CD\min} P_C\right)}{C\left(\gamma_{C\min} P_C\right)}. \tag{32}$$

Therefore, we have the following admission control strategy for the D2D link: if (31) and (32) are satisfied, the D2D link is admitted to the network and the cooperative scheme is employed; otherwise, the D2D link is blocked by the network.

Remark 2. According to (27) – (30), the time allocation factor and the power allocation factor at CT are determined only by the channel quality related to CT, and are independent with the channel quality of the D2D link. This result is in consistence with the assumption that the cellular system has no information about the D2D channel quality.

IV. Optimal Contract Design Under Continuous Private Information

Based on the cooperative relaying scheme, in this section we study the optimal contract design problem under the scenario where the private information γ_D is continuous.

A. Contract Formulation

Suppose γ_D is a continuous random variable distributed according to a probability density function $f\left(\gamma_D\right)$ on the interval $\Upsilon = \left[\underline{\gamma_D}, \overline{\gamma_D}\right]$. By writing $T\left(P_D\left(\gamma_D\right)\right)$ as $T\left(\gamma_D\right)$ for simplicity, we can express the power-payment contract as $\{\left(P_D\left(\gamma_D\right), T\left(\gamma_D\right)\right), \forall \gamma_D \in \Upsilon\}$. Following from (30), the D2D data rate with private information γ_D can be written as

$$R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right) = \left(1 - \delta^*\right) C\left(\beta^* \gamma_D P_D\left(\gamma_D\right)\right). \tag{33}$$

According to the revelation principle [29], the constraint (4) of Problem OCD is equivalent to a set of individual rationality and incentive compatibility constraints:

- Individual rationality (IR) constraint:

$$R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right) - \mu T\left(\gamma_D\right) \geq 0, \quad \forall \gamma_D \in \Upsilon. \tag{34}$$

- Incentive compatibility (IC) constraint:

$$\begin{aligned} &R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right) - \mu T\left(\gamma_D\right)\\ &\geq R_D^*\left(P_D\left(\hat{\gamma_D}\right), \gamma_D\right) - \mu T\left(\hat{\gamma_D}\right), \quad \forall \gamma_D, \hat{\gamma_D} \in \Upsilon. \end{aligned} \tag{35}$$

For the D2D link with any private information $\gamma_D \in \Upsilon$, the IR constraint ensures that it receives a non-negative utility by accepting the contract, and the IC constraint ensures that it receives the maximum utility by accepting the contract item designed for its private information γ_D. By defining the individual rationality constraint for γ_D (IR_{γ_D}) as

$$R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right) - \mu T\left(\gamma_D\right) \geq 0, \tag{36}$$

and the incentive compatibility constraint for γ_D (IC_{γ_D}) as

$$\begin{aligned} &R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right) - \mu T\left(\gamma_D\right)\\ &\geq R_D^*\left(P_D\left(\hat{\gamma_D}\right), \gamma_D\right) - \mu T\left(\hat{\gamma_D}\right), \quad \forall \hat{\gamma_D} \in \Upsilon, \end{aligned} \tag{37}$$

we can also write the IR and IC constraints as $\left\{\text{IR}_{\gamma_D}, \forall \gamma_D \in \Upsilon\right\}$ and $\left\{\text{IC}_{\gamma_D}, \forall \gamma_D \in \Upsilon\right\}$ respectively.

Therefore, the **o**ptimal **c**ontract **d**esign problem under **c**ontinuous private information can be formulated as

Problem OCD-C:

$$\max_{P_D(\gamma_D), T(\gamma_D)} \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} \left[T\left(\gamma_D\right) - \pi\left(P_D\left(\gamma_D\right)\right)\right] f\left(\gamma_D\right) \mathrm{d}\gamma_D \tag{38}$$

$$\text{s.t. (IR), (IC).} \tag{39}$$

B. Problem Reduction

In the following, we study how to reduce the problem without losing optimality by reducing the IR and IC constraints to a more tractable set of constraints.

First, we analyze the reduction of the IR constraint by the following two lemmas.

Lemma 3: At the optimal solution of Problem OCD-C, all constraints IR_{γ_D} *for* $\gamma_D > \underline{\gamma_D}$ *are redundant.*

Proof: Indeed, given $\mathrm{IR}_{\underline{\gamma_D}}$ and IC, all constraints IR_{γ_D} for $\gamma_D > \underline{\gamma_D}$ will be automatically satisfied: $\forall\, \gamma_D > \underline{\gamma_D}$,

$$\begin{aligned} & R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right) - \mu T\left(\gamma_D\right) \\ & \overset{(a)}{\geq} R_D^*\left(P_D\left(\underline{\gamma_D}\right), \gamma_D\right) - \mu T\left(\underline{\gamma_D}\right) \\ & \overset{(b)}{\geq} R_D^*\left(P_D\left(\underline{\gamma_D}\right), \underline{\gamma_D}\right) - \mu T\left(\underline{\gamma_D}\right) \\ & \overset{(c)}{\geq} 0, \end{aligned} \tag{40}$$

where (a) follows from the constraint IC, (b) comes from the fact that $\frac{\partial R_D^*(P_D(\gamma_D),\gamma_D)}{\partial \gamma_D} > 0$, and (c) follows from the constraint $\mathrm{IR}_{\underline{\gamma_D}}$. ■

Lemma 4: At the optimal solution of Problem OCD-C, the constraint $IR_{\underline{\gamma_D}}$ binds.

Proof: Assume to the contrary that $\mathrm{IR}_{\underline{\gamma_D}}$ does not bind. Then, we can increase $T\left(\gamma_D\right)$ by a small $\epsilon > 0$ for all $\gamma_D \in \Upsilon$, which would preserve $\mathrm{IR}_{\underline{\gamma_D}}$, not affect IC, and improve the maximand. Thus, we have a contradiction, and $\mathrm{IR}_{\underline{\gamma_D}}$ binds at the optimal solution. ■

Lemmas 3 and 4 imply that the IR constraint of Problem OCD-C can be reduced to the following constraint:

$$R_D^*\left(P_D\left(\underline{\gamma_D}\right), \underline{\gamma_D}\right) - \mu T\left(\underline{\gamma_D}\right) = 0. \tag{41}$$

Next, we analyze the reduction of the IC constraint by the following three lemmas.

Lemma 5: At the optimal solution of Problem OCD-C, $P_D\left(\gamma_D\right)$ is monotonically increasing in γ_D, i.e., $\frac{\mathrm{d}P_D(\gamma_D)}{\mathrm{d}\gamma_D} \geq 0$.

Proof: See Appendix A. ■

Lemma 6: With $\frac{\mathrm{d}P_D(\gamma_D)}{\mathrm{d}\gamma_D} \geq 0$, the constraint IC_{γ_D} is equivalent to the set of constraints $\left\{IC^{<}_{\gamma_D}, IC^{>}_{\gamma_D}, IC^{=}_{\gamma_D}\right\}$, where $IC^{<}_{\gamma_D}$:

$$\frac{\partial R_D^*\left(P_D\left(\hat{\gamma_D}\right), \gamma_D\right)}{\partial P_D\left(\hat{\gamma_D}\right)} \frac{\mathrm{d}P_D\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} \geq \mu \frac{\mathrm{d}T\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}}, \quad \forall \hat{\gamma_D} < \gamma_D, \tag{42}$$

$IC^{>}_{\gamma_D}$:

$$\frac{\partial R_D^*\left(P_D\left(\hat{\gamma_D}\right), \gamma_D\right)}{\partial P_D\left(\hat{\gamma_D}\right)} \frac{\mathrm{d}P_D\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} \leq \mu \frac{\mathrm{d}T\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}}, \quad \forall \hat{\gamma_D} > \gamma_D, \tag{43}$$

$IC^{=}_{\gamma_D}$:

$$\frac{\partial R_D^*\left(P_D\left(\hat{\gamma_D}\right), \gamma_D\right)}{\partial P_D\left(\hat{\gamma_D}\right)} \frac{\mathrm{d}P_D\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} = \mu \frac{\mathrm{d}T\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}}, \quad \hat{\gamma_D} = \gamma_D. \tag{44}$$

Proof: See Appendix B. ■

Lemma 7: At the optimal solution of Problem OCD-C, all constraints $IC^{<}_{\gamma_D}$ and $IC^{>}_{\gamma_D}$ are redundant.

Proof: See Appendix C. ■

Lemmas 5-7 imply that the IC constraint of Problem OCD-C can be reduced to the following constraints:

$$\begin{cases} \mu \frac{\mathrm{d}T(\gamma_D)}{\mathrm{d}\gamma_D} = \frac{\partial R_D^*(P_D(\gamma_D),\gamma_D)}{\partial P_D(\gamma_D)} \frac{\mathrm{d}P_D\left(\gamma_D\right)}{\mathrm{d}\gamma_D}, & (45) \\ \frac{\mathrm{d}P_D\left(\gamma_D\right)}{\mathrm{d}\gamma_D} \geq 0. & (46) \end{cases}$$

Thus, the constraint (39) can be replaced by (41), (45) and (46). Moreover, by (45), we get

$$\begin{aligned} \mu \frac{\mathrm{d}T\left(\gamma_D\right)}{\mathrm{d}\gamma_D} &= \frac{\partial R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right)}{\partial P_D\left(\gamma_D\right)} \frac{\mathrm{d}P_D\left(\gamma_D\right)}{\mathrm{d}\gamma_D} \\ &= \frac{\mathrm{d}R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right)}{\mathrm{d}\gamma_D} - \frac{\partial R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right)}{\partial \gamma_D}, \end{aligned} \tag{47}$$

or integrating,

$$\begin{aligned} \mu \int_{\underline{\gamma_D}}^{\gamma_D} \mathrm{d}T\left(x\right) = \int_{\underline{\gamma_D}}^{\gamma_D} \mathrm{d}R_D^*\left(P_D\left(x\right), x\right) \cdots \\ \cdots - \int_{\underline{\gamma_D}}^{\gamma_D} \frac{\partial R_D^*\left(P_D\left(x\right), x\right)}{\partial x} \mathrm{d}x. \end{aligned} \tag{48}$$

Substituting (41) into (48), we have

$$\mu T\left(\gamma_D\right) = R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right) - \int_{\underline{\gamma_D}}^{\gamma_D} \frac{\partial R_D^*\left(P_D\left(x\right), x\right)}{\partial x} \mathrm{d}x. \tag{49}$$

Incorporating (49) into (38) and performing integral operations (see Appendix D), we can rewrite the objective function of Problem OCD-C as[2]

$$\begin{aligned} & \mu \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} \left[T\left(\gamma_D\right) - \pi\left(P_D\left(\gamma_D\right)\right)\right] f\left(\gamma_D\right) \mathrm{d}\gamma_D \\ & = \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} \Big\{\left[R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right) - \mu\pi\left(P_D\left(\gamma_D\right)\right)\right] f\left(\gamma_D\right) \cdots \\ & \quad \cdots - \frac{\partial R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right)}{\partial \gamma_D} \left[1 - F\left(\gamma_D\right)\right]\Big\} \mathrm{d}\gamma_D, \end{aligned} \tag{50}$$

where $F\left(\gamma_D\right)$ is the cumulative distribution function of γ_D.

Therefore, the problem OCD-C can be equivalently rewritten as the following reduced problem:

Problem OCD-C-r:

$$\max_{P_D(\gamma_D)} \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} L\left(P_D\left(\gamma_D\right), \gamma_D\right) \mathrm{d}\gamma_D \tag{51}$$

$$\text{s.t.}\ \frac{\mathrm{d}P_D\left(\gamma_D\right)}{\mathrm{d}\gamma_D} \geq 0, \tag{52}$$

where $L\left(P_D\left(\gamma_D\right), \gamma_D\right) = f\left(\gamma_D\right)\left[R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right) \cdots \right.$ $\left. \cdots - \mu\pi\left(P_D\left(\gamma_D\right)\right)\right] - \left[1 - F\left(\gamma_D\right)\right] \frac{\partial R_D^*(P_D(\gamma_D),\gamma_D)}{\partial \gamma_D}$.

C. Optimal Solution

Now, we proceed to solve the Problem OCD-C-r. By introducing a new variable $w\left(\gamma_D\right)$, we can rewrite the Problem OCD-C-r as

$$\max_{P_D(\gamma_D)} \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} L\left(P_D\left(\gamma_D\right), \gamma_D\right) \mathrm{d}\gamma_D \tag{53}$$

$$\text{s.t.}\ \frac{\mathrm{d}P_D\left(\gamma_D\right)}{\mathrm{d}\gamma_D} = w\left(\gamma_D\right), \tag{54}$$

$$w\left(\gamma_D\right) \geq 0, \tag{55}$$

[2]Note that multiplying the objective function by a constant μ does not change the optimality of the problem.

which appears in the format of a continuous-time optimal control problem, with state variable $P_D(\gamma_D)$ and control variable $w(\gamma_D)$. We can solve this problem using tools from continuous-time optimal control theory [38, Sec. 3.3.1].

The Hamiltonian for this problem is

$$\begin{aligned} &H\left(P_D(\gamma_D), w(\gamma_D), \lambda(\gamma_D)\right) \\ &\quad = L\left(P_D(\gamma_D), \gamma_D\right) + \lambda(\gamma_D)\, w(\gamma_D), \end{aligned} \tag{56}$$

where $\lambda(\gamma_D)$ is the multiplier function associated with $w(\gamma_D)$. Since $\frac{\partial^2}{\partial w^2(\gamma_D)} H\left(P_D(\gamma_D), w(\gamma_D), \lambda(\gamma_D)\right) = 0$, Pontryagin's maximum principle can provide the following two necessary and sufficient conditions for an optimum $\left\{P_D^*(\gamma_D), w^*(\gamma_D), \lambda^*(\gamma_D)\right\}$:

$$\begin{aligned} &H\left(P_D^*(\gamma_D), w^*(\gamma_D), \lambda^*(\gamma_D)\right) \\ &\quad \geq H\left(P_D^*(\gamma_D), w(\gamma_D), \lambda^*(\gamma_D)\right), \end{aligned} \tag{57}$$

and

$$\begin{aligned} \frac{\mathrm{d}\lambda^*(\gamma_D)}{\mathrm{d}\gamma_D} &= -\frac{\partial H\left(P_D^*(\gamma_D), w^*(\gamma_D), \lambda^*(\gamma_D)\right)}{\partial P_D^*(\gamma_D)} \\ &= -\frac{\partial L\left(P_D^*(\gamma_D), \gamma_D\right)}{\partial P_D^*(\gamma_D)}. \end{aligned} \tag{58}$$

Next, we solve the Problem OCD-C-r by considering the following three cases.

(i) There exists *unique* optimal state variable $P_D^*(\gamma_D)$.

Since $w^*(\gamma_D) \geq 0$, we discuss the solutions for $w^*(\gamma_D) > 0$ and $w^*(\gamma_D) = 0$ respectively. If $w^*(\gamma_D) > 0$, then $P_D^*(\gamma_D)$ should satisfy $\frac{\partial L\left(P_D^*(\gamma_D), \gamma_D\right)}{\partial P_D^*(\gamma_D)} = 0$. By solving this equation, we can obtain $P_D^*(\gamma_D)$. If $w^*(\gamma_D) = 0$, we assume there are N_Λ intervals over which $w^*(\gamma_D) = 0$, and denote the j-th interval by $\Lambda_j = \left(\gamma_{Dj}^-, \gamma_{Dj}^+\right)$,[3] $j = 1, ..., N_\Lambda$. According to (54), $P_D^*(\gamma_D)$ is a constant over each Λ_j. Following from the continuity of $P_D^*(\gamma_D)$, we have $P_D^*(\gamma_D) = P_D^*\left(\gamma_{Dj}^-\right) = P_D^*\left(\gamma_{Dj}^+\right)$, $\forall \gamma_D \in \Lambda_j$, where $P_D^*\left(\gamma_{Dj}^-\right)$ and $P_D^*\left(\gamma_{Dj}^+\right)$ are obtained in case $w^*(\gamma_D) > 0$.

By the above analysis, we have obtained $P_D^*(\gamma_D)$ for $w^*(\gamma_D) > 0$ and $w^*(\gamma_D) = 0$. It remains to determine γ_{Dj}^- and γ_{Dj}^+ for $j = 1, ..., N_\Lambda$. For γ_{Dj}^-, $w^*\left(\gamma_{Dj}^-\right) > 0$, and thus (57) requires $\frac{\partial H\left(P_D^*\left(\gamma_{Dj}^-\right), w^*\left(\gamma_{Dj}^-\right), \lambda^*\left(\gamma_{Dj}^-\right)\right)}{\partial w^*\left(\gamma_{Dj}^-\right)} = 0$, implying $\lambda^*\left(\gamma_{Dj}^-\right) = 0$. Similarly, $\lambda^*\left(\gamma_{Dj}^+\right) = 0$. Then by (58), we get $\lambda^*\left(\gamma_{Dj}^-\right) - \lambda^*\left(\gamma_{Dj}^+\right) = \int_{\gamma_{Dj}^-}^{\gamma_{Dj}^+} \frac{\partial L\left(P_D^*(\gamma_D), \gamma_D\right)}{\partial P_D^*(\gamma_D)} \mathrm{d}\gamma_D = 0$. Combining this equation and $P_D^*\left(\gamma_{Dj}^-\right) = P_D^*\left(\gamma_{Dj}^+\right)$, we have two equations with two unknowns, allowing us to determine γ_{Dj}^- and γ_{Dj}^+. Algorithm 1 summarizes the procedure of deriving the optimal solution for the *unique* optimal state variable case.

(ii) There exist *multiple* optimal state variables $P_D^{*m}(\gamma_D)$, $m = 1, ..., M$.

[3]Note that $\left\{\Lambda_j,\ j = 1, ..., N_\Lambda\right\}$ satisfies: (1) $\Lambda_j \cap \Lambda_i = \emptyset$, $\forall j \neq i$, (2) $w^*(\gamma_D) = 0$, $\forall \gamma_D \in \left\{\Lambda_j\right\}$ (3) $w^*(\gamma_D) > 0$, $\forall \gamma_D \notin \left\{\Lambda_j\right\}$.

Algorithm 1 Optimal Contract Design Under Continuous γ_D (Unique $\widetilde{P_D}(\gamma_D)$ Case)

1: Solve the following equation and obtain the solution $\widetilde{P_D}(\gamma_D)$:

$$\frac{\partial L\left(P_D(\gamma_D), \gamma_D\right)}{\partial P_D(\gamma_D)} = 0. \tag{59}$$

2: Solve the following set of simultaneous equations and obtain all intervals $\Lambda_j = \left(\gamma_{Dj}^-, \gamma_{Dj}^+\right)$:

$$\begin{cases} \int_{\gamma_{Dj}^-}^{\gamma_{Dj}^+} \frac{\partial L\left(\widetilde{P_D}\left(\gamma_{Dj}^+\right), \gamma_D\right)}{\partial \widetilde{P_D}\left(\gamma_{Dj}^+\right)} \mathrm{d}\gamma_D = 0, & (60) \\ \widetilde{P_D}\left(\gamma_{Dj}^-\right) = \widetilde{P_D}\left(\gamma_{Dj}^+\right). & (61) \end{cases}$$

3: Calculate the optimal contract $\left\{P_D^*(\gamma_D), T^*(\gamma_D)\right\}$:

$$P_D^*(\gamma_D) = \begin{cases} \widetilde{P_D}\left(\gamma_{Dj}^+\right), & \text{if } \gamma_D \in \Lambda_j, \\ \widetilde{P_D}(\gamma_D), & \text{otherwise.} \end{cases} \tag{62}$$

$$\begin{aligned} T^*(\gamma_D) = \frac{1}{\mu}\Bigg[& R_D^*\left(P_D^*(\gamma_D), \gamma_D\right) \cdots \\ & \cdots - \int_{\underline{\gamma_D}}^{\gamma_D} \frac{\partial R_D^*\left(P_D^*(x), x\right)}{\partial x} \mathrm{d}x \Bigg]. \end{aligned} \tag{63}$$

In this case, we can calculate $P_D^{*m}(\gamma_D)$ for each m according to steps 2-3 in Algorithm 1, and then get $P_D^*(\gamma_D) = \arg\max_{P_D^{*m}(\gamma_D)} \left\{L\left(P_D^{*m}(\gamma_D), \gamma_D\right), m = 1, ..., M\right\}$. Then, we can obtain $T^*(\gamma_D)$ according to (63).

(iii) There exists *no* optimal state variable $P_D^*(\gamma_D)$.

In this case, the optimal contract dose not exist, and the network can block the D2D link.

V. Optimal Contract Design under Discrete Private Information

Based on the cooperative relaying scheme, in this section we study the optimal contract design problem under the scenario where the private information γ_D is discrete.

A. Contract Formulation

Suppose γ_D is a discrete random variable that can be denoted by $\gamma_{D,i}$, $i = 1, \ldots, N$. Without loss of generality, we assume $\gamma_{D,1} < \gamma_{D,2} < \ldots < \gamma_{D,N}$. Denote the probability of $\gamma_{D,i}$ by p_i, then $\sum_{i=1}^N p_i = 1$. By writing $P_D\left(\gamma_{D,i}\right)$ and $T\left(\gamma_{D,i}\right)$ as $P_{D,i}$ and T_i for simplicity, we can express the power-payment contract as $\left\{\left(P_{D,i}, T_i\right), i = 1, \ldots, N\right\}$. Following from (30), the D2D data rate with private information $\gamma_{D,i}$ can be written as

$$R_D^*\left(P_{D,i}, \gamma_{D,i}\right) = \left(1 - \delta^*\right) C\left(\beta^* \gamma_{D,i} P_{D,i}\right). \tag{64}$$

Similar to the continuous-γ_D model, the constraint (4) is equivalent to the following set of constraints:

- Individual rationality (IR) constraint:

$$R_D^*\left(P_{D,i}, \gamma_{D,i}\right) - \mu T_i \geq 0, \quad \forall i = 1, \ldots, N. \tag{65}$$

- Incentive compatibility (IC) constraint:

$$R_D^*\left(P_{D,i},\gamma_{D,i}\right)-\mu T_i \geq R_D^*\left(P_{D,j},\gamma_{D,i}\right)-\mu T_j,\quad \forall i,j=1,\ldots,N. \tag{66}$$

Defining the individual rationality constraint for $\gamma_{D,i}$ (IR$_i$) as

$$R_D^*\left(P_{D,i},\gamma_{D,i}\right)-\mu T_i \geq 0, \tag{67}$$

and the incentive compatibility constraint for $\gamma_{D,i}$ (IC$_i$) as

$$R_D^*\left(P_{D,i},\gamma_{D,i}\right)-\mu T_i \geq R_D^*\left(P_{D,j},\gamma_{D,i}\right)-\mu T_j,\quad \forall j=1,\ldots,N, \tag{68}$$

we can also write the IR and IC constraints as $\{\text{IR}_i,\ i=1,\ldots,N\}$ and $\{\text{IC}_i,\ i=1,\ldots,N\}$ respectively.

Therefore, the **o**ptimal **c**ontract **d**esign problem under **d**iscrete private information can be formulated as

Problem OCD-D:

$$\max_{\{P_{D,i},T_i\}} \sum_{i=1}^{N} p_i\left[T_i-\pi\left(P_{D,i}\right)\right] \tag{69}$$

$$\text{s.t. (IR), (IC).} \tag{70}$$

B. Problem Reduction

First, we analyze the reduction of the IR constraint by the following lemma.

Lemma 8: At the optimal solution of Problem OCD-D, the constraints IR$_2$ $\sim$ IR$_N$ are redundant, whereas the constraint IR$_1$ binds.

Proof: Following from the proofs of Lemmas 3 and 4, and substituting γ_D and $\underline{\gamma_D}$ with $\gamma_{D,i}$ and $\gamma_{D,1}$ respectively, we complete the proof. ■

Lemma 8 implies that the IR constraint of Problem OCD-D can be reduced to the following constraint:

$$R_D^*\left(P_{D,1},\gamma_{D,1}\right)-\mu T_1 = 0. \tag{71}$$

Next, we analyze the reduction of the IC constraint by the following three lemmas.

Lemma 9: At the optimal solution of Problem OCD-D, P_D is monotonically increasing in γ_D, i.e., $P_{D,i}\leq P_{D,j},\ \forall i<j$.

Proof: Following from the proof of Lemma 5, and substituting γ_D and $\gamma_D+\sigma$ with $\gamma_{D,i}$ and $\gamma_{D,j}$ respectively, we complete the proof. ■

Lemma 10: With the monotonicity of P_D, the constraint IC$_i$ for $2\leq i\leq N-1$ is equivalent to $\left\{IC_i^{<}, IC_i^{>}\right\}$, where IC$_i^{<}$:

$$R_D^*\left(P_{D,i},\gamma_{D,i}\right)-R_D^*\left(P_{D,i-1},\gamma_{D,i}\right)\geq \mu\left(T_i-T_{i-1}\right), \tag{72}$$

IC$_i^{>}$:

$$R_D^*\left(P_{D,i+1},\gamma_{D,i}\right)-R_D^*\left(P_{D,i},\gamma_{D,i}\right)\leq \mu\left(T_{i+1}-T_i\right). \tag{73}$$

Proof: See Appendix E. ■

Using similar approaches, we can also show that IC$_1$ is equivalent to IC$_1^{>}$ and IC$_N$ is equivalent to IC$_N^{<}$, where IC$_1^{>}$ and IC$_N^{<}$ can be obtained by letting $i=1$ in (73) and $i=N$ in (72) respectively.

Lemma 11: At the optimal solution of Problem OCD-D, all constraints IC$_i^{<}$ $(2\leq i\leq N)$ bind, whereas all constraints IC$_j^{>}$ $(1\leq j\leq N-1)$ are redundant.

Proof: See Appendix F. ■

Lemmas 9-11 imply that the IC constraint of Problem OCD-D can be reduced to the following constraints:

$$\begin{cases} R_D^*\left(P_{D,i},\gamma_{D,i}\right)-R_D^*\left(P_{D,i-1},\gamma_{D,i}\right) \\ =\mu T_i-\mu T_{i-1},\quad \forall i\geq 2, & (74)\\ P_{D,i}\leq P_{D,j},\quad \forall i<j. & (75)\end{cases}$$

By incorporating (71) and (74) into the objective function (69), we can equivalently rewrite the problem OCD-D as the following reduced problem:

Problem OCD-D-r:

$$\max_{\{P_{D,i}\}} \sum_{i=1}^{N} L\left(P_{D,i}\right) \tag{76}$$

$$\text{s.t. } P_{D,i}\leq P_{D,j},\ \forall i<j \tag{77}$$

where $L\left(P_{D,i}\right)=\sigma_i R_D^*\left(P_{D,i},\gamma_{D,i}\right)-\nu_i R_D^*\left(P_{D,i},\gamma_{D,i+1}\right)-\mu p_i\pi\left(P_{D,i}\right)$, with $\sigma_i=\sum_{k=i}^{N}p_k$, $\nu_i=1-\sum_{k=1}^{i}p_k$.

C. Optimal Solution

Now, we proceed to solve the Problem OCD-D-r. By introducing a new variable w_i, we can rewrite the Problem OCD-D-r as

$$\max_{\{P_{D,i}\}} \sum_{i=1}^{N} L\left(P_{D,i}\right) \tag{78}$$

$$\text{s.t. } P_{D,i+1}-P_{D,i}=w_i,\quad \forall 1\leq i\leq N-1 \tag{79}$$

$$w_i\geq 0,\quad \forall 1\leq i\leq N-1 \tag{80}$$

which appears in the format of a discrete-time optimal control problem, with state variable $P_{D,i}$ and control variable w_i. We can solve this problem using tools from discrete-time optimal control theory [38, Sec. 3.3.3].

The Hamiltonian for this problem is

$$H\left(P_{D,i},w_i,\lambda_{i+1}\right)=L\left(P_{D,i}\right)+\lambda_{i+1}w_i, \tag{81}$$

where λ_{i+1} is the multiplier associated with w_i. Pontryagin's maximum principle can provide the following two necessary and sufficient conditions for an optimum $\left\{P_{D,i}^*,w_i^*,\lambda_{i+1}^*\right\}$:

$$H\left(P_{D,i}^*,w_i^*,\lambda_{i+1}^*\right)\geq H\left(P_{D,i}^*,w_i,\lambda_{i+1}^*\right), \tag{82}$$

and

$$\lambda_{i+1}^*-\lambda_i^*=-\frac{\partial H\left(P_{D,i}^*,w_i^*,\lambda_{i+1}^*\right)}{\partial P_{D,i}^*}=-\frac{\mathrm{d}L\left(P_{D,i}^*\right)}{\mathrm{d}P_{D,i}^*}. \tag{83}$$

Next, we solve the Problem OCD-D-r by considering the following three cases.

(i) There exists *unique* optimal state variable sequence $\left\{P_{D,i}^*\right\}$.

Since $w_i^*\geq 0$, we next discuss the cases $w_i^*>0$ and $w_i^*=0$ $(1\leq i\leq N-1)$ respectively. If $w_i^*>0$, then $\frac{\mathrm{d}L\left(P_{D,i}^*\right)}{\mathrm{d}P_{D,i}^*}=0$. By solving this equation, we can obtain $\left\{P_{D,i}^*\right\}$. If $w_i^*=0$,

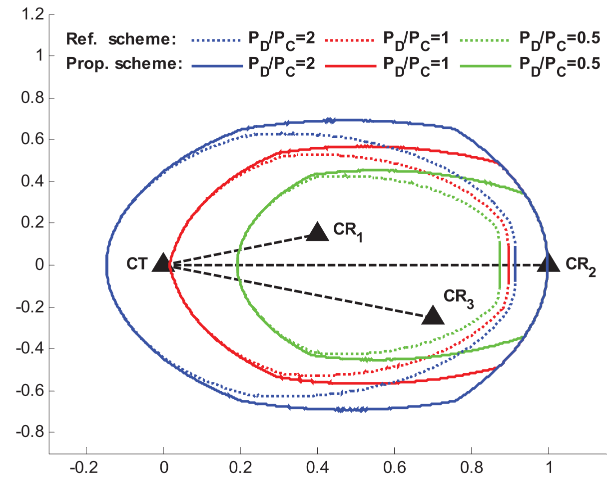


Fig. 3. Admission regions for the cooperative relaying schemes.

we assume there are N_Γ intervals over which $w_i^* = 0$, and denote the j-th interval by $\Gamma_j = \left\{\gamma_{D,m_j}, \ldots, \gamma_{D,n_j}\right\}$,[4] $j = 1, \ldots, N_\Gamma$. According to (79), $P^*_{D,i}$ is a constant over each Γ_j and $P^*_{D,i} = P^*_{D,n_j+1}$, $\forall \gamma_{D,i} \in \Gamma_j$, where P^*_{D,n_j+1} is obtained in case $w_i^* > 0$.

By the above analysis, we have obtained $P^*_{D,i}$ for both the cases $w_i^* > 0$ and $w_i^* = 0$. It remains to determine m_j and n_j for $j = 1, \ldots, N_\Gamma$. For γ_{D,n_j+1}, since $w^*_{n_j+1} > 0$, the condition (82) requires $\frac{\partial H\left(P^*_{D,n_j+1}, w^*_{n_j+1}, \lambda^*_{n_j+2}\right)}{\partial w^*_{n_j+1}} = 0$, which implies $\lambda^*_{n_j+2} = 0$. According to (83), $\lambda^*_{n_j+2} - \lambda^*_{n_j+1} = -\frac{\mathrm{d}L\left(P^*_{D,n_j+1}\right)}{\mathrm{d}P^*_{D,n_j+1}} = 0$. Thereby, $\lambda^*_{n_j+1} = 0$. Similarly, $\lambda^*_{m_j} = 0$. Moreover, the condition (83) yields that $\lambda^*_{n_j+1} - \lambda^*_{m_j} = -\sum_{i=m_j}^{n_j} \frac{\mathrm{d}L\left(P^*_{D,i}\right)}{\mathrm{d}P^*_{D,i}}$. Therefore, $\sum_{i=m_j}^{n_j} \frac{\mathrm{d}L\left(P^*_{D,i}\right)}{\mathrm{d}P^*_{D,i}} = 0$. Combining this equation and $\widetilde{P}_{D,n_j+1} \geq \widetilde{P}_{D,m_j-1}$, we have one equation and one inequation with two unknowns. Then by extend j-th infeasible interval,[5] we can determine m_j, n_j. Algorithm 2 summarizes the procedure of deriving the optimal solution for the unique optimal state variable sequence case.

(ii) There exist *multiple* optimal state variable sequences $\{P^*_{D,i}\}_m$, $m = 1, \ldots, M$.

In this case, we can calculate $\{P^*_{D,i}\}_m$ for each m according to steps 2-3 in Algorithm 2, and then get $\left\{P^*_{D,i}\right\} = \arg\max_{\left\{P^*_{D,i}\right\}_m} \left\{\sum_{i=1}^N L\left(\left\{P^*_{D,i}\right\}_m\right), m = 1, \ldots, M\right\}$. Then, we can obtain $\left\{T_i^*\right\}$ according to (88).

(iii) There exists *no* optimal state variable sequence $\left\{P^*_{D,i}\right\}$.

In this case, the optimal contract dose not exist, and the network can block the D2D link.

[4]Note that $\left\{\Gamma_j, j = 1, \ldots, N_\Gamma\right\}$ satisfies: (1) $\Gamma_j \cap \Gamma_i = \emptyset$, $\forall j \neq i$, (2) $w_i^* = 0$, $\forall \gamma_{D,i} \in \left\{\Gamma_j\right\}$, (3) $w_i^* > 0$, $\forall \gamma_{D,i} \notin \left\{\Gamma_j\right\}$.

[5]An interval $\left\{P_{D,\hat{m}_j}, P_{D,\hat{m}_j+1}, \ldots, P_{D,\hat{n}_j}\right\}$ is an infeasible interval if $P_{D,\hat{m}_j} \geq \ldots \geq P_{D,\hat{n}_j}$, $P_{D,\hat{m}_j-1} \leq P_{D,\hat{m}_j}$, $P_{D,\hat{n}_j} \leq P_{D,\hat{n}_j+1}$.

Algorithm 2 Optimal Contract Design Under Discrete γ_D (Unique $\left\{\widetilde{P}_{D,i}\right\}$ Case)

1: Solve the following equation and obtain the solution $\left\{\widetilde{P}_{D,i}\right\}$:

$$\frac{\mathrm{d}L\left(P_{D,i}\right)}{\mathrm{d}P_{D,i}} = 0. \tag{84}$$

2: Obtain all intervals $\Gamma_j = \left\{\gamma_{D,m_j}, \ldots, \gamma_{D,n_j}\right\}$ using the following method:

Find j-th infeasible interval $\left\{\gamma_{D,\hat{m}_j}, \ldots, \gamma_{D,\hat{n}_j}\right\}$, then

for $t \leftarrow \hat{n}_j : \hat{m}_{j+1}$ **do**

for $k \leftarrow \hat{m}_j : \hat{n}_{j-1}$ **do**

if the following equation and inequation hold,

$$\begin{cases} \sum_{i=k}^{t} \frac{\mathrm{d}L\left(\widetilde{P}_{D,i}\right)}{\mathrm{d}\widetilde{P}_{D,i}} \big|_{\widetilde{P}_{D,i}=\widetilde{P}_{D,t}} = 0, & (85) \\ \widetilde{P}_{D,t} \geq \widetilde{P}_{D,k-1}. & (86) \end{cases}$$

then $\gamma_{D,m_j} \leftarrow \gamma_{D,k}$, $\gamma_{D,n_j} \leftarrow \gamma_{D,t}$, **break**;

end if

end for

end for

3: Calculate the optimal contract $\left\{P^*_{D,i}, T_i^*\right\}$:

$$P^*_{D,i} = \begin{cases} \widetilde{P}_{D,n_j+1}, & \text{if } \gamma_{D,i} \in \Gamma_j, \\ \widetilde{P}_{D,i}, & \text{otherwise.} \end{cases} \tag{87}$$

$$T_i^* = \begin{cases} \frac{1}{\mu} R_D^*\left(P^*_{D,1}, \gamma_{D,1}\right), & i = 1, \\ \frac{1}{\mu}\left[R_D^*\left(P^*_{D,i}, \gamma_{D,i}\right) - \sum_{k=1}^{i-1} h_k\right], & i \geq 2, \end{cases} \tag{88}$$

where $h_k = R_D^*\left(P^*_{D,k}, \gamma_{D,k+1}\right) - R_D^*\left(P^*_{D,k}, \gamma_{D,k}\right)$.

VI. Simulation Results

Here we provide some numerical results on the performance of the proposed cooperative relaying scheme and optimal contracts. We consider a system topology in an X-Y plane, where CT is located at point $(0, 0)$, and CR_1, CR_2, CR_3 are located at points $(0.4, 0.15)$, $(1, 0)$ and $(0.7, -0.25)$ respectively.

A. Performance Evaluation of Cooperative Relaying Scheme

Assume $\gamma_i = \frac{1}{d_i^\nu}$, where d_i is the distance of corresponding link and ν $(= 4)$ is the path loss exponent. We consider the scenario where DT is close to DR and thus γ_{CDT} approximately equals γ_{CDR}. To compare the performance of the proposed cooperative relaying scheme, we present a reference scheme in which the cellular transmitter does not employ superposition coding. The reference scheme can be viewed as a special case of the proposed scheme where $\alpha^* = 1$. Figure 3 shows the admission regions of DT. We can see that the admission region for the proposed scheme is larger than that for the reference scheme, which indicates that more D2D links can benefit from the proposed cooperative relaying scheme.

Letting DT move on the X axis, we can get the numerical results shown in Figs. 4 and 5. Figure 4 compares the D2D data rates of the two schemes. We can observe that the proposed scheme can achieve a higher D2D data rate than the reference scheme. This is because in the proposed scheme, the cellular

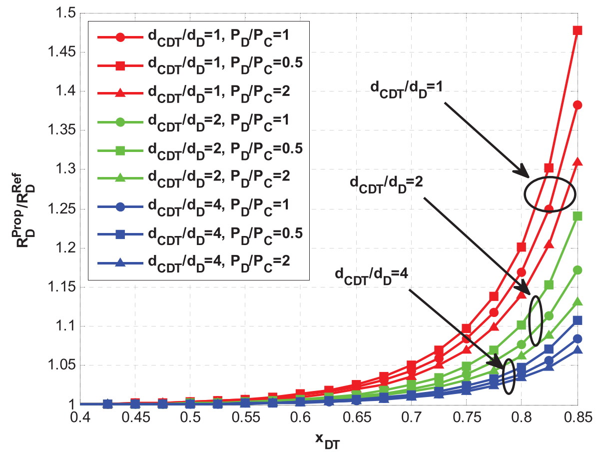


Fig. 4. D2D data rate.

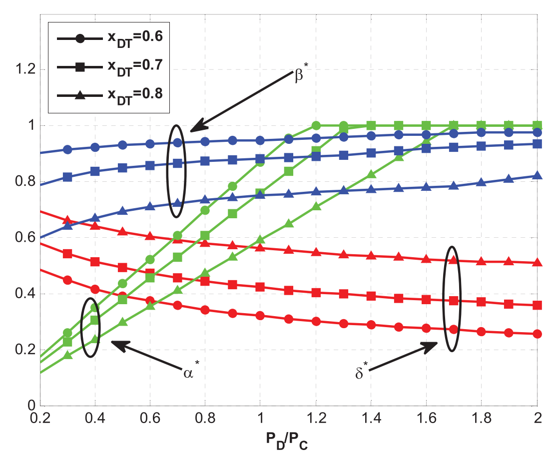
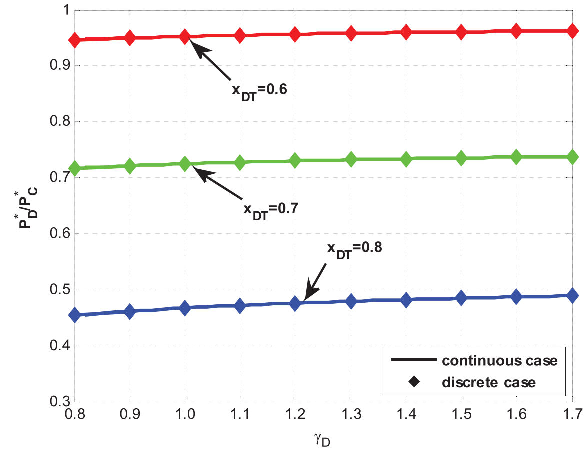
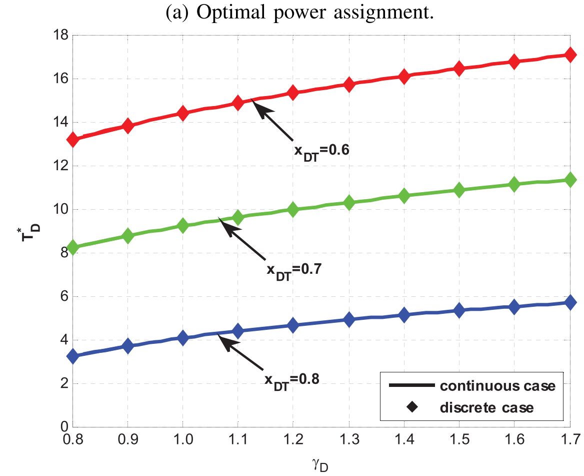


Fig. 5. Optimal resource allocation factors.



(a) Optimal power assignment.



(b) Optimal payment assignment.

Fig. 6. Optimal contract for example 1.

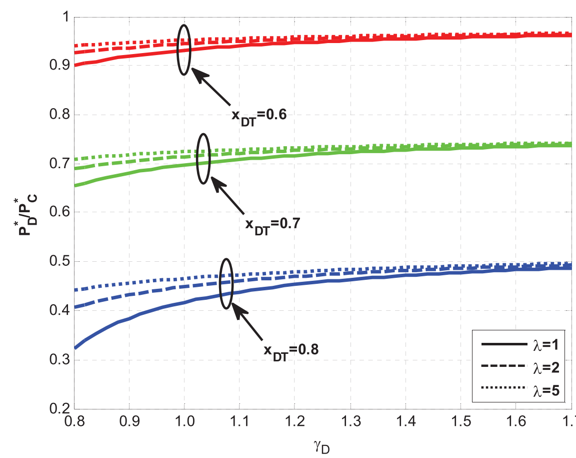


Fig. 7. Optimal contract for example 2.

signal is created by superposition coding and only a part of the cellular signal is relayed by the D2D transmitter, thereby leaving more transmission power for D2D communication. From Fig. 4 we can also observe that the ratio of D2D data rates of the two schemes becomes larger with increasing d_{CDT}, and meanwhile a smaller P_D leads to a higher ratio. This result indicates that when DT is far away from CT or has a small transmission power, the proposed scheme can provide more transmission opportunities for the D2D link. Figure 5 presents the values of α^*, β^*, δ^* under the scenario where $d_D = \frac{1}{2} d_{CDT}$. We can observe that when DT has a larger transmission power, the first phase takes less time and DT can allocate a larger share of transmission power to D2D signal.

B. Performance Evaluation of Optimal Contracts

We provide several examples to show the performance of the optimal contracts.

Example 1 (Continuous- and Discrete-γ_D Cases): We assume γ_D follows a uniform distribution over the interval [0.8, 1.7] and the cost function is linear. Figure 6 presents the power and payment assignments in the optimal contracts. Through Fig. 6(a) we can see that the optimal power increases as γ_D increases, which is in consistence with Lemma 5. Through Fig. 6(b) we can see that the optimal payment is also monotonically increasing in γ_D. These results imply that in the optimal contract, the cellular system offers a higher power-payment bundle to the D2D link with higher channel quality. Figure 6 also shows that the power-payment bundle becomes smaller when DT moves forward on the X axis. This indicates that the optimal power and payment that maximize $L\left(P_D\left(\gamma_D\right), \gamma_D\right)$ decrease as γ_{CDT} decreases.

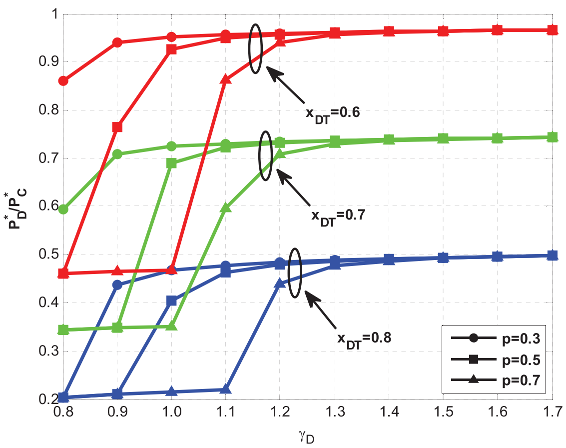


Fig. 8. Optimal contract for example 3.

Example 2 (Continuous-γ_D Case): We assume γ_D follows an exponential distribution with mean λ (i.e., Rayleigh fading environment), where $\lambda = 1,\ 2,\ 5$. We also consider a linear cost function. Figure 7 presents the power assignment in the optimal contract. Through the figure we can see that when the probabilities of higher channel qualities become lower (i.e., λ increases), the cellular system tends to allow a larger D2D transmission power.

Example 3 (Discrete-γ_D Case): We assume $\gamma_{D,i} = 0.8 + 0.1 \times i$, $i = 0, 1, \ldots, 9$, and i follows a binomial distribution with parameters $n = 10$ and p, where $p = 0.3,\ 0.5,\ 0.7$. We also consider a linear cost function. Figure 8 presents the power assignment in the optimal contract. Through the figure we can see that when the probabilities of higher channel qualities become larger (i.e., p increases), the cellular system tends to allow a smaller D2D transmission power.

VII. Conclusion

In this paper, we study the spectrum sharing problem for D2D-enabled cellular networks, and propose a contract-based cooperative spectrum sharing mechanism to exploit transmission opportunities for the D2D links and meanwhile achieve the maximum profit of the cellular links. We first design a superposition coding-based cooperative relaying scheme that can maximize the data rate of the D2D links without deteriorating the performance of the cellular links, and then propose a contract-theoretic framework to model the spectrum trading process based on the cooperative relaying scheme, and derive the optimal power-payment contracts for the cellular links under both the cases that the private information of the D2D links is continuous and discrete using tools from continuous- and discrete-time optimal control theories respectively.

Interesting topics for further research include employing other signal processing techniques like Maximum Ratio Combining (MRC) and Successive Interference Cancellation (SIC) to further improve the data rate of D2D link, considering the hidden action problem (i.e., the cellular system does not know whether the D2D link allocates transmission power "honestly") for the spectrum trading contract, and incorporating mode selection and link scheduling schemes into the contract-based spectrum sharing mechanism.

Appendix A
Proof of Lemma 5

$\forall \gamma_D \in \left(\underline{\gamma_D}, \overline{\gamma_D}\right)$, consider $\sigma > 0$ and $\gamma_D + \sigma \in \left[\underline{\gamma_D}, \overline{\gamma_D}\right]$. According to the IC constraint, we have

$$R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right) - \mu T\left(\gamma_D\right) \geq R_D^*\left(P_D\left(\gamma_D + \sigma\right), \gamma_D\right) - \mu T\left(\gamma_D + \sigma\right), \quad (89)$$

and

$$R_D^*\left(P_D\left(\gamma_D + \sigma\right), \gamma_D + \sigma\right) - \mu T\left(\gamma_D + \sigma\right) \geq R_D^*\left(P_D\left(\gamma_D\right), \gamma_D + \sigma\right) - \mu T\left(\gamma_D\right). \quad (90)$$

Combining (89) and (90), we get

$$R_D^*\left(P_D\left(\gamma_D + \sigma\right), \gamma_D + \sigma\right) - R_D^*\left(P_D\left(\gamma_D\right), \gamma_D + \sigma\right) \geq R_D^*\left(P_D\left(\gamma_D + \sigma\right), \gamma_D\right) - R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right). \quad (91)$$

The left side of the above inequation is equal to $\int_{P_D(\gamma_D)}^{P_D(\gamma_D+\sigma)} \frac{\partial R_D^*(P_D(\gamma_D),\gamma_D+\sigma)}{\partial P_D(\gamma_D)} \mathrm{d}P_D\left(\gamma_D\right)$, and the right side is equal to $\int_{P_D(\gamma_D)}^{P_D(\gamma_D+\sigma)} \frac{\partial R_D^*(P_D(\gamma_D),\gamma_D)}{\partial P_D(\gamma_D)} \mathrm{d}P_D\left(\gamma_D\right)$. Thus, we have

$$\int_{P_D(\gamma_D)}^{P_D(\gamma_D+\sigma)} \left[\frac{\partial R_D^*\left(P_D\left(\gamma_D\right), \gamma_D + \sigma\right)}{\partial P_D\left(\gamma_D\right)} \cdots \cdots - \frac{\partial R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right)}{\partial P_D\left(\gamma_D\right)}\right] \mathrm{d}P_D\left(\gamma_D\right) \geq 0, \quad (92)$$

and thereby

$$\int_{P_D(\gamma_D)}^{P_D(\gamma_D+\sigma)} \int_{\gamma_D}^{\gamma_D+\sigma} \frac{\partial^2 R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right)}{\partial P_D\left(\gamma_D\right) \partial \gamma_D} \mathrm{d}\gamma_D \mathrm{d}P_D\left(\gamma_D\right) \geq 0. \quad (93)$$

Then, since $\frac{\partial^2 R_D^*(P_D(\gamma_D),\gamma_D)}{\partial P_D(\gamma_D)\partial \gamma_D} > 0$, we get

$$P_D\left(\gamma_D + \sigma\right) \geq P_D\left(\gamma_D\right). \quad (94)$$

Therefore,

$$\frac{\mathrm{d}P_D\left(\gamma_D\right)}{\mathrm{d}\gamma_D} = \lim_{\sigma \to 0} \frac{P_D\left(\gamma_D + \sigma\right) - P_D\left(\gamma_D\right)}{\sigma} \geq 0. \quad (95)$$

Appendix B
Proof of Lemma 6

The sufficiency of the set of constraints can be straightforwardly proved. In the following, we show the necessity of the set of constraints: For any γ_D,

(i) $\forall \hat{\gamma_D} \in \left(\underline{\gamma_D}, \gamma_D\right)$, consider $\sigma > 0$ and $\hat{\gamma_D} - \sigma \geq \underline{\gamma_D}$. According to IC_{γ_D},

$$R_D^*\left(P_D\left(\hat{\gamma_D}\right), \hat{\gamma_D}\right) - \mu T\left(\hat{\gamma_D}\right) \geq R_D^*\left(P_D\left(\hat{\gamma_D} - \sigma\right), \hat{\gamma_D}\right) - \mu T\left(\hat{\gamma_D} - \sigma\right). \quad (96)$$

By using the fundamental theorem of calculus, we have

$$\begin{aligned} &R_D^*\left(P_D\left(\hat{\gamma_D}\right), \hat{\gamma_D}\right) - R_D^*\left(P_D\left(\hat{\gamma_D} - \sigma\right), \hat{\gamma_D}\right) \\ &= \int_{P_D(\hat{\gamma_D}-\sigma)}^{P_D(\hat{\gamma_D})} \frac{\partial R_D^*\left(P_D\left(\gamma_D\right), \hat{\gamma_D}\right)}{\partial P_D\left(\gamma_D\right)} \mathrm{d}P_D\left(\gamma_D\right) \\ &\overset{(a)}{\leq} \int_{P_D(\hat{\gamma_D}-\sigma)}^{P_D(\hat{\gamma_D})} \frac{\partial R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right)}{\partial P_D\left(\gamma_D\right)} \mathrm{d}P_D\left(\gamma_D\right) \\ &= R_D^*\left(P_D\left(\hat{\gamma_D}\right), \gamma_D\right) - R_D^*\left(P_D\left(\hat{\gamma_D} - \sigma\right), \gamma_D\right), \end{aligned} \quad (97)$$

where (a) comes from the fact that $\frac{\partial^2 R_D^*(P_D(\gamma_D),\gamma_D)}{\partial P_D(\gamma_D)\partial \gamma_D} > 0$ and $\frac{\mathrm{d}P_D(\gamma_D)}{\mathrm{d}\gamma_D} \geq 0$. Combining (96) and (97), we get

$$R_D^*\left(P_D\left(\hat{\gamma_D}\right), \gamma_D\right) - R_D^*\left(P_D\left(\hat{\gamma_D} - \sigma\right), \gamma_D\right) \geq \mu\left[T\left(\hat{\gamma_D}\right) - T\left(\hat{\gamma_D} - \sigma\right)\right]. \quad (98)$$

Taking the limit as $\sigma \to 0$ on both sides of (98) yields

$$\lim_{\sigma\to 0} \frac{R_D^*\left(P_D\left(\hat{\gamma_D}\right), \gamma_D\right) - R_D^*\left(P_D\left(\hat{\gamma_D} - \sigma\right), \gamma_D\right)}{\sigma} \geq \mu \lim_{\sigma\to 0} \frac{T\left(\hat{\gamma_D}\right) - T\left(\hat{\gamma_D} - \sigma\right)}{\sigma}, \quad (99)$$

which implies

$$\frac{\partial R_D^*\left(P_D\left(\hat{\gamma_D}\right), \gamma_D\right)}{\partial P_D\left(\hat{\gamma_D}\right)} \frac{\mathrm{d}P_D\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} \geq \mu \frac{\mathrm{d}T\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}}. \quad (100)$$

(ii) $\forall\, \hat{\gamma_D} \in (\gamma_D, \overline{\gamma_D})$, consider $\sigma > 0$ and $\hat{\gamma_D} + \sigma \leq \overline{\gamma_D}$. According to IC_{γ_D},

$$R_D^*\left(P_D\left(\hat{\gamma_D}\right), \hat{\gamma_D}\right) - \mu T\left(\hat{\gamma_D}\right) \geq R_D^*\left(P_D\left(\hat{\gamma_D} + \sigma\right), \hat{\gamma_D}\right) - \mu T\left(\hat{\gamma_D} + \sigma\right). \quad (101)$$

By using the fundamental theorem of calculus, we have

$$\begin{aligned} &R_D^*\left(P_D\left(\hat{\gamma_D} + \sigma\right), \hat{\gamma_D}\right) - R_D^*\left(P_D\left(\hat{\gamma_D}\right), \hat{\gamma_D}\right) \\ &= \int_{P_D(\hat{\gamma_D})}^{P_D(\hat{\gamma_D}+\sigma)} \frac{\partial R_D^*\left(P_D\left(\gamma_D\right), \hat{\gamma_D}\right)}{\partial P_D\left(\gamma_D\right)} \mathrm{d}P_D\left(\gamma_D\right) \\ &\geq \int_{P_D(\hat{\gamma_D})}^{P_D(\hat{\gamma_D}+\sigma)} \frac{\partial R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right)}{\partial P_D\left(\gamma_D\right)} \mathrm{d}P_D\left(\gamma_D\right) \\ &= R_D^*\left(P_D\left(\hat{\gamma_D} + \sigma\right), \gamma_D\right) - R_D^*\left(P_D\left(\hat{\gamma_D}\right), \gamma_D\right). \quad (102) \end{aligned}$$

Combining (101) and (102), we get

$$R_D^*\left(P_D\left(\hat{\gamma_D} + \sigma\right), \gamma_D\right) - R_D^*\left(P_D\left(\hat{\gamma_D}\right), \gamma_D\right) \leq \mu\left[T\left(\hat{\gamma_D} + \sigma\right) - T\left(\hat{\gamma_D}\right)\right]. \quad (103)$$

Then by taking the limit as $\sigma \to 0$ on both sides of (103), we have

$$\frac{\partial R_D^*\left(P_D\left(\hat{\gamma_D}\right), \gamma_D\right)}{\partial P_D\left(\hat{\gamma_D}\right)} \frac{\mathrm{d}P_D\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} \leq \mu \frac{\mathrm{d}T\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}}. \quad (104)$$

(iii) Let $g\left(\hat{\gamma_D}\right) = \frac{\partial R_D^*\left(P_D\left(\hat{\gamma_D}\right), \gamma_D\right)}{\partial P_D\left(\hat{\gamma_D}\right)} \frac{\mathrm{d}P_D\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} - \mu \frac{\mathrm{d}T\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}}$. Since $g\left(\hat{\gamma_D}\right)$ is differentiable at $\hat{\gamma_D} = \gamma_D$, $g\left(\hat{\gamma_D}\right)$ is continuous at $\hat{\gamma_D} = \gamma_D$. According to the results of $(i)\,(ii)$, $g\left(\hat{\gamma_D}\right) \geq 0$ when $\hat{\gamma_D} < \gamma_D$ and $g\left(\hat{\gamma_D}\right) \leq 0$ when $\hat{\gamma_D} > \gamma_D$. Therefore, $g\left(\hat{\gamma_D}\right) = 0$ when $\hat{\gamma_D} = \gamma_D$, i.e.,

$$g\left(\hat{\gamma_D}\right)\Big|_{\hat{\gamma_D}=\gamma_D} = \frac{\partial R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right)}{\partial P_D\left(\gamma_D\right)} \frac{\mathrm{d}P_D\left(\gamma_D\right)}{\mathrm{d}\gamma_D} \cdots - \mu \frac{\mathrm{d}T\left(\gamma_D\right)}{\mathrm{d}\gamma_D} = 0. \quad (105)$$

Combining (i), (ii) and (iii), we complete the proof of the necessity.

Appendix C
Proof of Lemma 7

We shall show that, given $\mathrm{IC}_{\gamma_D}^{=}$, all constraints $\mathrm{IC}_{\gamma_D}^{<}$ and $\mathrm{IC}_{\gamma_D}^{>}$ will be automatically satisfied: $\forall\, \hat{\gamma_D}, \gamma_D \in \left[\underline{\gamma_D}, \overline{\gamma_D}\right]$, according to $\mathrm{IC}_{\gamma_D}^{=}$, we have

$$\frac{\partial R_D^*\left(P_D\left(\hat{\gamma_D}\right), \hat{\gamma_D}\right)}{\partial P_D\left(\hat{\gamma_D}\right)} \frac{\mathrm{d}P_D\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} - \mu \frac{\mathrm{d}T\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} = 0. \quad (106)$$

Thus, when $\hat{\gamma_D} < \gamma_D$,

$$\begin{aligned} g\left(\hat{\gamma_D}\right) &= \frac{\partial R_D^*\left(P_D\left(\hat{\gamma_D}\right), \gamma_D\right)}{\partial P_D\left(\hat{\gamma_D}\right)} \frac{\mathrm{d}P_D\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} - \mu \frac{\mathrm{d}T\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} \\ &\overset{(a)}{\geq} \frac{\partial R_D^*\left(P_D\left(\hat{\gamma_D}\right), \hat{\gamma_D}\right)}{\partial P_D\left(\hat{\gamma_D}\right)} \frac{\mathrm{d}P_D\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} - \mu \frac{\mathrm{d}T\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} \\ &= 0, \end{aligned} \quad (107)$$

and when $\hat{\gamma_D} > \gamma_D$,

$$\begin{aligned} g\left(\hat{\gamma_D}\right) &= \frac{\partial R_D^*\left(P_D\left(\hat{\gamma_D}\right), \gamma_D\right)}{\partial P_D\left(\hat{\gamma_D}\right)} \frac{\mathrm{d}P_D\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} - \mu \frac{\mathrm{d}T\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} \\ &\overset{(b)}{\leq} \frac{\partial R_D^*\left(P_D\left(\hat{\gamma_D}\right), \hat{\gamma_D}\right)}{\partial P_D\left(\hat{\gamma_D}\right)} \frac{\mathrm{d}P_D\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} - \mu \frac{\mathrm{d}T\left(\hat{\gamma_D}\right)}{\mathrm{d}\hat{\gamma_D}} \\ &= 0, \end{aligned} \quad (108)$$

where (a), (b) come from the fact that $\frac{\partial^2 R_D^*(P_D(\gamma_D),\gamma_D)}{\partial P_D(\gamma_D)\partial \gamma_D} > 0$ and $\frac{\mathrm{d}P_D(\gamma_D)}{\mathrm{d}\gamma_D} \geq 0$.

Appendix D
Derivation of Eq. (50)

Substituting (49) into (38), we have

$$\begin{aligned} &\mu \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} \left[T\left(\gamma_D\right) - \pi\left(P_D\left(\gamma_D\right)\right)\right] f\left(\gamma_D\right) \mathrm{d}\gamma_D \\ &= \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} \left[R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right) - \int_{\underline{\gamma_D}}^{\gamma_D} \frac{\partial R_D^*\left(P_D\left(x\right), x\right)}{\partial x} \mathrm{d}x \cdots \right. \\ &\qquad \left. \cdots - \mu\pi\left(P_D\left(\gamma_D\right)\right)\right] f\left(\gamma_D\right) \mathrm{d}\gamma_D \\ &= \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} \left[R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right) - \mu\pi\left(P_D\left(\gamma_D\right)\right)\right] f\left(\gamma_D\right) \mathrm{d}\gamma_D \cdots \\ &\qquad \cdots - \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} \left[\int_{\underline{\gamma_D}}^{\gamma_D} \frac{\partial R_D^*\left(P_D\left(x\right), x\right)}{\partial x} \mathrm{d}x\right] f\left(\gamma_D\right) \mathrm{d}\gamma_D. \end{aligned} \quad (109)$$

Solving the second integral through integration by parts,

$$\begin{aligned}
&\int_{\underline{\gamma_D}}^{\overline{\gamma_D}} \left[\int_{\underline{\gamma_D}}^{\gamma_D} \frac{\partial R_D^* \left(P_D\left(x\right), x\right)}{\partial x} \mathrm{d}x \right] f\left(\gamma_D\right) \mathrm{d}\gamma_D \\
&= \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} \int_{\underline{\gamma_D}}^{\gamma_D} \frac{\partial R_D^* \left(P_D\left(x\right), x\right)}{\partial x} \mathrm{d}x \, \mathrm{d}F\left(\gamma_D\right) \\
&= \left(\int_{\underline{\gamma_D}}^{\gamma_D} \frac{\partial R_D^* \left(P_D\left(x\right), x\right)}{\partial x} \mathrm{d}x \right) F\left(\gamma_D\right) \Bigg|_{\underline{\gamma_D}}^{\overline{\gamma_D}} \cdots \\
&\quad \cdots - \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} F\left(\gamma_D\right) \mathrm{d} \left(\int_{\underline{\gamma_D}}^{\gamma_D} \frac{\partial R_D^* \left(P_D\left(x\right), x\right)}{\partial x} \mathrm{d}x \right) \\
&= \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} \frac{\partial R_D^* \left(P_D\left(x\right), x\right)}{\partial x} \mathrm{d}x \cdots \\
&\quad \cdots - \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} F\left(\gamma_D\right) \frac{\partial R_D^* \left(P_D\left(\gamma_D\right), \gamma_D\right)}{\partial \gamma_D} \mathrm{d}\gamma_D \\
&= \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} \frac{\partial R_D^* \left(P_D\left(\gamma_D\right), \gamma_D\right)}{\partial \gamma_D} \left[1 - F\left(\gamma_D\right)\right] \mathrm{d}\gamma_D, \qquad (110)
\end{aligned}$$

and substituting (110) into (109), we can rewrite the objective function as

$$\begin{aligned}
&\mu \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} \left[T\left(\gamma_D\right) - \pi\left(P_D\left(\gamma_D\right)\right)\right] f\left(\gamma_D\right) \mathrm{d}\gamma_D \\
&= \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} \left[R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right) - \mu\pi\left(P_D\left(\gamma_D\right)\right)\right] f\left(\gamma_D\right) \mathrm{d}\gamma_D \cdots \\
&\quad \cdots - \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} \frac{\partial R_D^* \left(P_D\left(\gamma_D\right), \gamma_D\right)}{\partial \gamma_D} \left[1 - F\left(\gamma_D\right)\right] \mathrm{d}\gamma_D \\
&= \int_{\underline{\gamma_D}}^{\overline{\gamma_D}} \Bigg\{ \left[R_D^*\left(P_D\left(\gamma_D\right), \gamma_D\right) - \mu\pi\left(P_D\left(\gamma_D\right)\right)\right] f\left(\gamma_D\right) \cdots \\
&\quad \cdots - \frac{\partial R_D^* \left(P_D\left(\gamma_D\right), \gamma_D\right)}{\partial \gamma_D} \left[1 - F\left(\gamma_D\right)\right] \Bigg\} \mathrm{d}\gamma_D. \qquad (111)
\end{aligned}$$

Appendix E
Proof of Lemma 10

(i) By substituting γ_D, $\hat{\gamma_D}$ and $\hat{\gamma_D} - \sigma$ in (98) with $\gamma_{D,i}$, $\gamma_{D,i-1}$ and $\gamma_{D,i-k}$ $(2 \leq k \leq i-1)$ respectively, we can get

$$R_D^*\left(P_{D,i-1}, \gamma_{D,i}\right) - R_D^*\left(P_{D,i-k}, \gamma_{D,i}\right) \geq \mu\left(T_{i-1} - T_{i-k}\right). \qquad (112)$$

Then, combining (112) and $\mathrm{IC}_i^<$ in (72), we have

$$R_D^*\left(P_{D,i}, \gamma_{D,i}\right) - R_D^*\left(P_{D,i-k}, \gamma_{D,i}\right) \geq \mu\left(T_i - T_{i-k}\right). \qquad (113)$$

This implies that, $\forall\, 2 \leq i \leq N-1$, if $\mathrm{IC}_i^<$ holds, then all other downward constraints in IC_i are satisfied.

(ii) Similarly, we can show that, $\forall\, 2 \leq i \leq N-1$, if $\mathrm{IC}_i^>$ holds, then all other upward constraints in IC_i are satisfied.

Combining (i) and (ii), we complete the proof.

Appendix F
Proof of Lemma 11

First, we show $\mathrm{IC}_i^<$ $(2 \leq i \leq N)$ binds. Assume to the contrary that $\mathrm{IC}_i^<$ does not bind. Then, we can increase all $T_k's$ $(k \geq i)$ by a small $\epsilon > 0$, which would preserve $\mathrm{IC}_i^<$, not affect $\mathrm{IC}_j^<$ $(j \neq i)$, and improve the maximand. Thus, we have a contradiction, and $\mathrm{IC}_i^<$ binds at the optimal solution.

Next, we show that, if all $\mathrm{IC}_i^<$ $(2 \leq i \leq N)$ bind, then the constraints $\mathrm{IC}_j^>$ $(1 \leq j \leq N-1)$ will be automatically satisfied: $\forall\, 1 \leq j \leq N-1$, since $\mathrm{IC}_{j+1}^<$ binds, we have

$$R_D^*\left(P_{D,j+1}, \gamma_{D,j+1}\right) - R_D^*\left(P_{D,j}, \gamma_{D,j+1}\right) = \mu\left(T_{j+1} - T_j\right). \qquad (114)$$

Considering that

$$\begin{aligned}
&R_D^*\left(P_{D,j+1}, \gamma_{D,j+1}\right) - R_D^*\left(P_{D,j}, \gamma_{D,j+1}\right) \\
&\quad = \int_{P_{D,j}}^{P_{D,j+1}} \frac{\partial R_D^*\left(P_{D,k}, \gamma_{D,j+1}\right)}{\partial P_{D,k}} \mathrm{d}P_{D,k} \\
&\quad \geq \int_{P_{D,j}}^{P_{D,j+1}} \frac{\partial R_D^*\left(P_{D,k}, \gamma_{D,j}\right)}{\partial P_{D,k}} \mathrm{d}P_{D,k} \\
&\quad \geq R_D^*\left(P_{D,j+1}, \gamma_{D,j}\right) - R_D^*\left(P_{D,j}, \gamma_{D,j}\right), \qquad (115)
\end{aligned}$$

together with (114), we get

$$R_D^*\left(P_{D,j+1}, \gamma_{D,j}\right) - R_D^*\left(P_{D,j}, \gamma_{D,j}\right) \leq \mu\left(T_{j+1} - T_j\right). \qquad (116)$$

Therefore, $\mathrm{IC}_j^>$ $(1 \leq j \leq N-1)$ are redundant.

References

[1] C. Ma, G. Sun, X. Tian, K. Ying, H. Yu, and X. Wang, "Cooperative relaying schemes for device-to-device communication underlaying cellular networks," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM)*, Dec. 2013, pp. 3890–3895.

[2] K. Doppler, M. Rinne, C. Wijting, C. B. Ribeiro, and K. Hugl, "Device-to-device communication as an underlay to LTE-advanced networks," *IEEE Commun. Mag.*, vol. 47, no. 12, pp. 42–49, Dec. 2009.

[3] A. Asadi, Q. Wang, and V. Mancuso, "A survey on device-to-device communication in cellular networks," *IEEE Commun. Surveys Tuts.*, vol. 16, no. 4, pp. 1801–1819, 4th Quart., 2014.

[4] L. Song, D. Niyato, Z. Han, and E. Hossain, *Wireless Device-to-Device Communications and Networks*. Cambridge, U.K.: Cambridge Univ. Press, 2015.

[5] H. Ma, L. Liu, A. Zhou, and D. Zhao, "On networking of Internet of things: Explorations and challenges," *IEEE Internet Things J.*, vol. 3, no. 4, pp. 441–452, Aug. 2016.

[6] *Technical Specification Group Services and System Aspects; Proximity-Based Services (ProSe), Rel-13*, document 3GPP TS 23.303, 2015.

[7] *Wireless World Initiative New Radio—WINNER+ D2.1, Preliminary WINNER+ System Concept*, Munich, Germany, 2009.

[8] G. Fodor *et al.*, "Design aspects of network assisted device-to-device communications," *IEEE Commun. Mag.*, vol. 50, no. 3, pp. 170–177, Mar. 2012.

[9] H. Min, W. Seo, J. Lee, S. Park, and D. Hong, "Reliability improvement using receive mode selection in the device-to-device uplink period underlaying cellular networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 2, pp. 413–418, Feb. 2011.

[10] B. Kaufman, J. Lilleberg, and B. Aazhang, "Spectrum sharing scheme between cellular users and ad-hoc device-to-device users," *IEEE Trans. Wireless Commun.*, vol. 12, no. 3, pp. 1038–1049, Mar. 2013.

[11] E. Zihan, K. W. Choi, and D. I. Kim, "Distributed random access scheme for collision avoidance in cellular device-to-device communication," *IEEE Trans. Wireless Commun.*, vol. 14, no. 7, pp. 3571–3585, Jul. 2015.

[12] L. Yang, W. Zhang, and S. Jin, "Interference alignment in device-to-device LAN underlaying cellular networks," *IEEE Trans. Wireless Commun.*, vol. 14, no. 7, pp. 3715–3723, Jul. 2015.

[13] H. Song, J. Y. Ryu, W. Choi, and R. Schober, "Joint power and rate control for device-to-device communications in cellular systems," *IEEE Trans. Wireless Commun.*, vol. 14, no. 10, pp. 5750–5762, Oct. 2015.

[14] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity. Part I. System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.

[15] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[16] I. Avram, N. Aerts, H. Bruneel, and M. Moeneclaey, "Quantize and forward cooperative communication: Channel parameter estimation," *IEEE Trans. Wireless Commun.*, vol. 11, no. 3, pp. 1167–1179, Mar. 2012.

[17] R. Zhang and L. Hanzo, "A unified treatment of superposition coding aided communications: Theory and practice," *IEEE Commun. Surveys Tuts.*, vol. 13, no. 3, pp. 503–520, 3rd Quart., 2011.

[18] P. Popovski and E. de Carvalho, "Improving the rates in wireless relay systems through superposition coding," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 4831–4836, Dec. 2008.

[19] Y. Han, A. Pandharipande, and S. H. Ting, "Cooperative decode-and-forward relaying for secondary spectrum access," *IEEE Trans. Wireless Commun.*, vol. 8, no. 10, pp. 4945–4950, Oct. 2009.

[20] E.-H. Shin and D. Kim, "Time and power allocation for collaborative primary-secondary transmission using superposition coding," *IEEE Commun. Lett.*, vol. 15, no. 2, pp. 196–198, Feb. 2011.

[21] J.-H. Wui and D. Kim, "Optimal power allocation between unicast and multicast messages in wireless relay-multicasting networks using superposition coding," *IEEE Commun. Lett.*, vol. 15, no. 11, pp. 1159–1161, Nov. 2011.

[22] S. Shalmashi and S. B. Slimane, "Cooperative device-to-device communications in the downlink of cellular networks," in *Proc. IEEE Wireless Commun. Netw. Conf.*, Istambul, Turkey, Apr. 2014, pp. 2265–2270.

[23] R. Yin, G. Yu, H. Zhang, Z. Zhang, and G. Y. Li, "Pricing-based interference coordination for D2D communications in cellular networks," *IEEE Trans. Wireless Commun.*, vol. 14, no. 3, pp. 1519–1532, Mar. 2015.

[24] Q. Wang, W. Wang, S. Jin, H. Zhu, and N. T. Zhang, "Quality-optimized joint source selection and power control for wireless multimedia D2D communication using stackelberg game," *IEEE Trans. Veh. Technol.*, vol. 64, no. 8, pp. 3755–3769, Aug. 2015.

[25] C. Xu *et al.*, "Efficiency resource allocation for device-to-device underlay communication systems," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 9, pp. 348–358, Sep. 2013.

[26] F. Wang, C. Xu, L. Song, and Z. Han, "Energy-efficient resource allocation for device-to-device underlay communication," *IEEE Trans. Wireless Commun.*, vol. 14, no. 4, pp. 2082–2092, Apr. 2015.

[27] C. Jiang, Y. Chen, Y. Gao, and K. J. R. Liu, "Joint spectrum sensing and access evolutionary game in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2470–2483, May 2013.

[28] C. Jiang, Y. Chen, and K. J. R. Liu, "Multi-channel sensing and access game: Bayesian social learning with negative network externality," *IEEE Trans. Wireless Commun.*, vol. 13, no. 4, pp. 2176–2188, Apr. 2014.

[29] P. Bolton and M. Dewatripont, *Contract Theory*. Cambridge, MA, USA: MIT Press, 2005.

[30] L. Gao, X. Wang, Y. Xu, and Q. Zhang, "Spectrum trading in cognitive radio networks: A contract-theoretic modeling approach," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 4, pp. 843–855, Apr. 2011.

[31] L. Gao, J. Huang, Y.-J. Chen, and B. Shou, "An integrated contract and auction design for secondary spectrum trading," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 3, pp. 581–592, Mar. 2013.

[32] D. M. Kalathil and R. Jain, "Spectrum sharing through contracts for cognitive radios," *IEEE Trans. Mobile Comput.*, vol. 12, no. 10, pp. 1999–2011, Oct. 2013.

[33] L. Duan, L. Gao, and J. Huang, "Cooperative spectrum sharing: A contract-based approach," *IEEE Trans. Mobile Comput.*, vol. 13, no. 1, pp. 174–187, Jan. 2014.

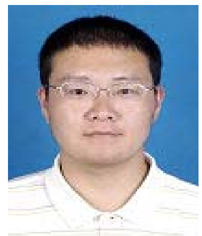
[34] Z. Hasan and V. K. Bhargava, "Relay selection for OFDM wireless systems under asymmetric information: A contract-theory based approach," *IEEE Trans. Wireless Commun.*, vol. 12, no. 8, pp. 3824–3837, Aug. 2013.

[35] M. Peng, X. Xie, Q. Hu, J. Zhang, and H. V. Poor, "Contract-based interference coordination in heterogeneous cloud radio access networks," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 6, pp. 1140–1153, Jun. 2015.

[36] Y. Li, J. Zhang, X. Gan, L. Fu, H. Yu, and X. Wang, "A contract-based incentive mechanism for delayed traffic offloading in cellular networks," *IEEE Trans. Wireless Commun.*, vol. 15, no. 8, pp. 5314–5327, Aug. 2016.

[37] Y. Zhang, L. Song, W. Saad, Z. Dawy, and Z. Han, "Contract-based incentive mechanisms for device-to-device communications in cellular ," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 10, pp. 2144–2155, Oct. 2015.

[38] D. P. Bertsekas, *Dynamic Programming and Optimal Control*. Belmont, MA, USA: Athena Scientific, 2005.



Chuan Ma received the B.E. degree in communication engineering from Sichuan University, China, in 2009, and the M.E. degree in information and communication engineering from Zhejiang University, China, in 2012. He is currently pursuing the Ph.D. degree with Shanghai Jiao Tong University, China. His research interests are in communication theory, device-to-device communication, vehicle-to-x communication, and heterogeneous networks.

Yuqing Li received the B.S. degree in electronic engineering from Xidian University, Xi'an, China, in 2014. She is currently pursuing the Ph.D. degree in electronic engineering with Shanghai Jiao Tong University, Shanghai, China. Her current research interests include network economics, social aware networks, heterogeneous cellular networks, and network security.

Hui Yu received the B.S. degree from the Department of Electrical Engineering, Tong Ji University in 1992, and the M.S. degree from the Department of Electronic Engineering, Shanghai Jiao Tong University in 1997. He is currently a Senior Engineer with the Department of Electronic Engineering, Shanghai Jiao Tong University. His research interests include mobile communications, software radio and cognitive radio, channel coding, and modulation for wireless communications.

Xiaoying Gan received the Ph.D. degree in electronic engineering from Shanghai Jiao Tong University, China, in 2006. She is with the Department of Electronic Engineering, Institute of Wireless Communication Technology, Shanghai Jiao Tong University, where she is currently an Associate Professor. From 2009 to 2010, she was a Visiting Researcher with the California Institute for Telecommunications and Information Technology, University of California San Diego, California. Her current research interests include network economics, social aware networks, heterogeneous cellular networks, cognitive networks, multi-user multi-channel access, and dynamic radio resource management.

Xinbing Wang received the B.S. degree from the Department of Automation, Shanghai Jiaotong University, Shanghai, China, in 1998, the M.S. degree from the Department of Computer Science and Technology, Tsinghua University, Beijing, China, in 2001, and the Ph.D. degree from the Department of electrical and Computer Engineering and the Department of Mathematics from North Carolina State University, Raleigh, in 2006. He is currently a Professor with the Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai, China. He has been an Associate Editor of the IEEE/ACM TRANSACTIONS ON NETWORKING and the IEEE TRANSACTIONS ON MOBILE COMPUTING, and a member of the Technical Program Committees of several conferences, including the ACM MobiCom 2012, the ACM MobiHoc from 2012 to 2014, the IEEE INFOCOM from 2009 to 2014.

Yong Ren received the B.S., M.S., and Ph.D. degrees from the Harbin Institute of Technology, China, in 1984, 1987, and 1994, respectively, all in electronic engineering. He was a Post Doctor with the Department of Electronics Engineering, Tsinghua University, China, from 1995 to 1997. He is currently a Professor with the Department of Electronics Engineering and the Director of the Complexity Engineered Systems Laboratory, Tsinghua University. He holds 12 patents and has authored or co-authored over 100 technical papers on the behavior of computer networks, P2P networks, and cognitive networks. He has serves as a Reviewer of the *IEICE Transactions on Communications*, the *Digital Signal Processing*, the *Chinese Physics Letters*, the *Chinese Journal of Electronics*, the *Chinese Journal of Computer Science & Technology*, and the *Chinese Journal of Aeronautics*. His current research interests include complex systems theory and its applications to the optimization and information sharing of the Internet, Internet of Things and ubiquitous networks, cognitive networks, and cyber-physical systems.

Jun (Jim) Xu received the Ph.D. degree in computer and information science from The Ohio State University in 2000. He is currently a Professor with the School of Computer Science, Georgia Tech. His current research interests include data streaming algorithms for the measurement and monitoring of computer networks, data structures and algorithms for highspeed routers, and social networking. He received the NSF CAREER award in 2003, the ACM Sigmetrics Best Student Paper Award in 2004, and IBM faculty awards in 2006 and 2008. He was named an ACM Distinguished Scientist in 2010.